SECTION A [30 MARKS]

ANSWER **ALL** QUESTION

Question 1 **[30]**

a) **For each question there are FOUR responses: A, B, C and D. Choose the corresponding letter of your response and CIRCLE it neatly. NO score will be awarded, if you circle more than ONE letter.**

i. What is the value of $\frac{1+\omega}{\omega+\omega^2}$, if ω is cube root of unity? 2

A 0

B 1

C ω

(D) ω^2

Criteria	**Marks**
Circles the correct option	**2**
Circles more than ONE alternative	**0**
Circles none of the alternatives	**0**

Answer: $\frac{1+\omega}{\omega+\omega^2} = \frac{-\omega^2}{-1} = \omega^2$

D. ω^2

ii. A school gardener wants to plant 10 different varieties of flowers around a circular walkway. How many different ways can the flowers be planted? 2

A 3628800

B 1814400

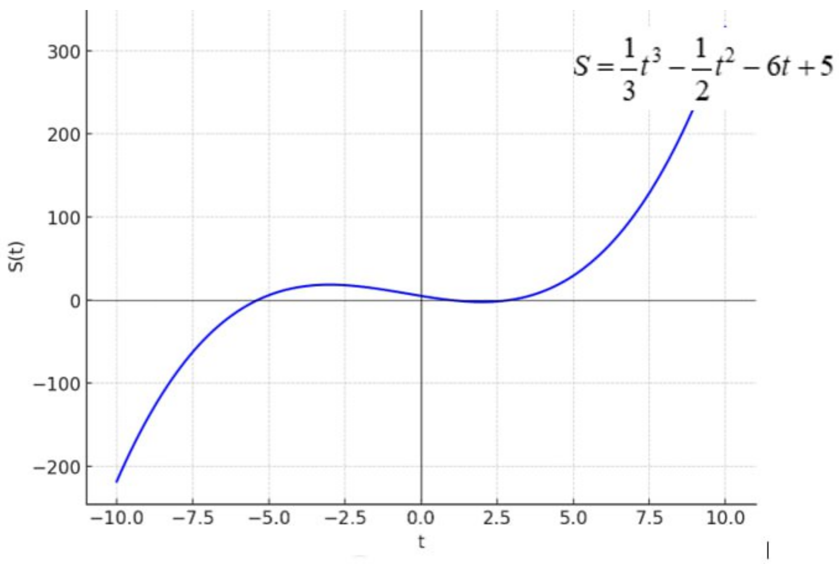
(C) 362880

D 181440

Answer: $(10-1)! = 9! = 362880$

C. 362880

iii. If A is a $3\times r$ matrix, B is a $5\times s$ matrix and the product of the matrix $AB=\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ then the value of r and s are		2
A $r=2$ and $s=5$. (B) $r=5$ and $s=2$. C $r=3$ and $s=2$. D $r=2$ and $s=3$. **Answer:** $[A]_{3\times r}\times[B]_{5\times s}=[AB]_{3\times 2}$ Hence $r=5$ and $s=2$. B. $r=5$ and $s=2$		
iv. If $\frac{d}{dx}(\log_e x)=\frac{1}{x}$, then $\frac{d}{dx}(\log_2 x)$ is		2
(A) $\frac{1}{x\log_e 2}$. B $\frac{1}{x\log_2 e}$. C $\frac{\log_e 2}{x}$. D $\frac{\log_2 e}{x}$. **Answer:** $\log_2 x=\frac{\log_e x}{\log_e 2}$ $\therefore \frac{d}{dx}(\log_2 x)=\frac{1}{\log_e 2}.\frac{1}{x}=\frac{1}{x\log_e 2}$ A. $\frac{1}{x\log_e 2}$		

v. Which of the following functions satisfies the property $\int_{-a}^{a} f(x)dx = 0$?		2
A $\lvert\sin x\rvert$ B $x^6 + 1$ (C) $x^2 \sin x$ D $(x^2+1)\cos x$ **Answer:** $f(x) = x^2 \sin x$, $f(-x) = (-x)^2 \sin(-x) = -x^2 \sin x$ Since it is an odd function, $\int_{-a}^{a} f(x)dx = 0$ C. $x^2 \sin x$		
vi. The graph represents the distance travelled by a particle in time t seconds. Study the graph and calculate the acceleration after 3 seconds.  		2
A $6\ ms^{-2}$ (B) $5\ ms^{-2}$ C $2.5\ ms^{-2}$ D $0\ ms^{-2}$		

Answer:

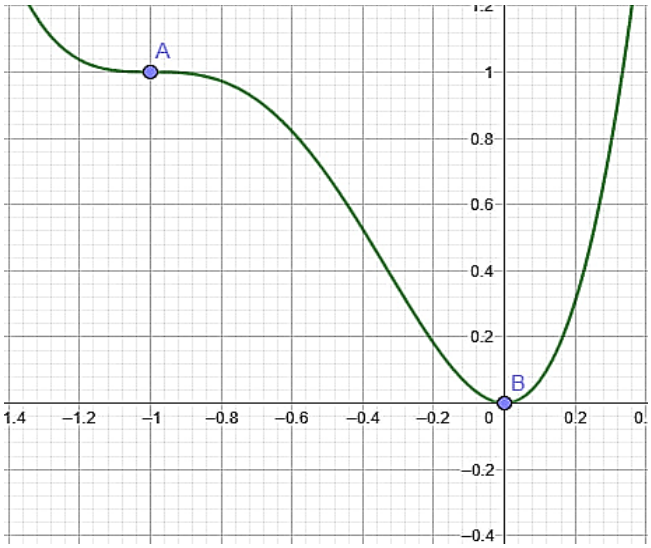
$$v \Rightarrow \frac{ds}{dt} = t^2 - t - 6, \quad a \Rightarrow \frac{d^2s}{dt^2} = 2t - 1$$

at $t = 3\sec$

$a = 2(3) - 1 = 5$

B. $5\,ms^{-2}$

vii. Which of the following is **NOT** true about points A and B in the given diagram. 2



A $\frac{dy}{dx} = 0$ at both A and B

B $\frac{d^2y}{dx^2} > 0$ at B

C $\frac{d^3y}{dx^3} \neq 0$ at A

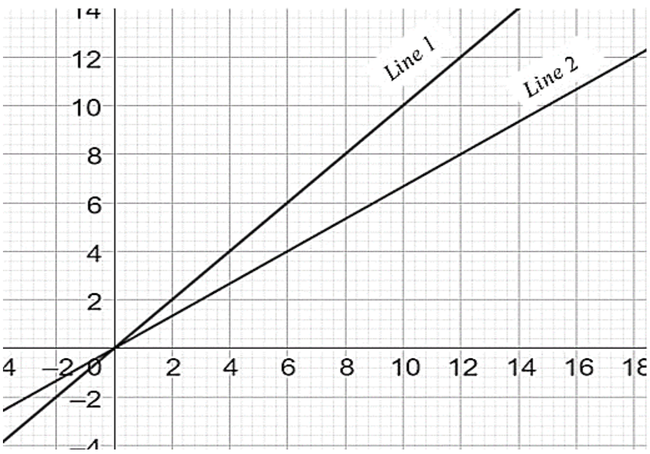
(D) $\frac{d^2y}{dx^2} \neq 0$ at A

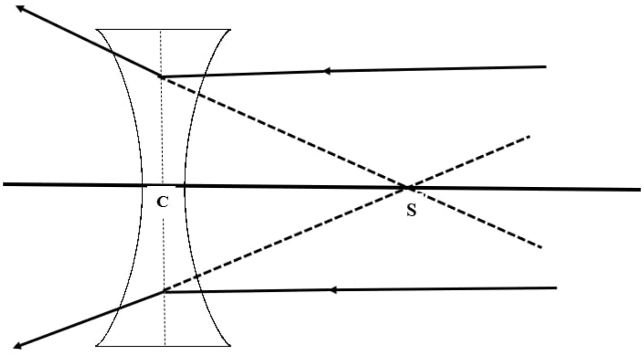
Answer: D.

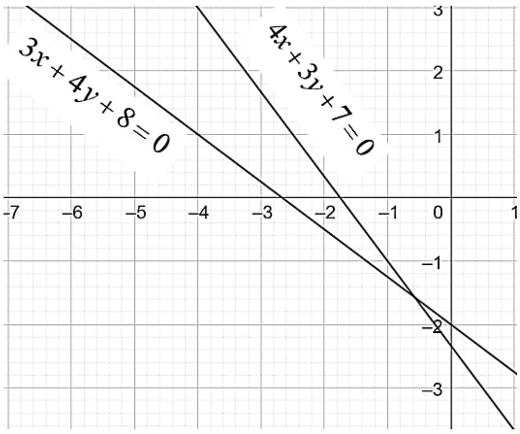
$\frac{d^2y}{dx^2} \neq 0$ at A since the point A is an inflexion point and only the odd differential coefficient should be non-zero

viii. Students are arguing about the order and degree of the differential equation $y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)$. The arguments are: *Pema: order 1 and degree 2* *Dema: order 1 and degree 3* *Kumar: order 2 and degree 2* *Gyem: order 2 and degree 3* Whose argument is correct?		2
A Pema (B) Dema C Kumar D Gyem **Answer:** $y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)$ $y = x\left(\frac{dy}{dx}\right)^2 + \frac{1}{\left(\frac{dy}{dx}\right)}$ $y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^3 + 1,$ order 1 and degree 3 B. Dema		
ix. Evaluate: $\sin\left[\cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{2}{3}\right)\right]$.		2
A $\frac{\pi}{2}$ B $\frac{2}{3}$ (C) 1 D 0 Answer: $\sin\left[\cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{2}{3}\right)\right]$ $= \sin\left[\frac{\pi}{2}\right], \quad \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$		

	$= 1$ C. 1	
x.	Karma and Bikash are participating in the national shotput championship. The throwing mark is at point $A(1,3,7)$. Karma threw the shotput and it landed at point $B(6,3,2)$. If the distance thrown by Bikash is two-third of the distance thrown by Karma, where will Bikash's shotput land?	2
	A (3, 3, 5) B $(11, 3, -3)$ C $\left(\frac{7}{2}, 3, \frac{9}{2}\right)$ (D) $\left(\frac{13}{3}, 3, \frac{11}{3}\right)$ **Answer:** $(x,y,z)=\left(\frac{2(6)+1(1)}{2+1},\frac{2(3)+1(3)}{2+1},\frac{2(2)+1(7)}{2+1}\right)$ $=\left(\frac{13}{3},3,\frac{11}{3}\right)$ D. $\left(\frac{13}{3},3,\frac{11}{3}\right)$	
xi.	Find the intercepts of the plane $6x-3y+2z-6=0$ on the axes.	2
	A $1,2,3$ B $6,3,2$ (C) $1,-2,3$ D $6,-3,2$ Answer: $6x-3y+2z-6=0$ $6x-3y+2z=6$ $\frac{6x}{6}-\frac{3y}{6}+\frac{2z}{6}=\frac{6}{6}$ $\frac{x}{1}-\frac{y}{2}+\frac{z}{3}=1$ Hence, the intercepts on the coordinates axes are 1,-2,3	

	C. 1,-2,3	
xii.	In the ongoing BoB football Premier league, the probability of winning the title by Paro FC is $\frac{3}{4}$ and Thimphu City is $\frac{1}{4}$. Which of the following events best describes the above situation?	2
	(A) Mutually exclusive, independent and exhaustive B Mutually exclusive, dependent and exhaustive C Mutually exclusive, dependent and not exhaustive D Not mutually exclusive, independent and exhaustive **Answer:** Event A: Paro FC winning title, $P(A)=\frac{3}{4}$ Event B: Thimphu City winning the title, $P(B)=\frac{1}{4}$ Since both clubs cannot win the title in one season, it is independent and mutually exclusive, and $P(A)+P(B)=1$; thus, it is an exhaustive event. A. Mutually exclusive, independent and exhaustive.	
xiii.	Combined equation of lines given in the graph is represented by $2x^2-5xy+3y^2=0$.What are the equations of line 1 and line 2?  	2
	(A) Line 1: $x-y=0$ Line 2: $2x-3y=0$ B Line 1: $2x-3y=0$ Line 2: $x-y=0$	

	C Line 1: $2x+3y=0$ Line 2: $x-y=0$ D Line 1: $x-y=0$ Line 2: $2x+3y=0$ **Answer:** $2x^2-5xy+3y^2=0$ $2x^2-3xy-2xy+3y^2=0$ $x(2x-3y)-y(2x-3y)=0$ $(2x-3y)(x-y)=0$ Hence, the lines are $2x-3y=0$ and $x-y=0$ A. Line 1: $x-y=0$ Line 2 : $2x-3y=0$	
xiv.	The given biconcave lens can be modeled by the equation $16x^2-9y^2=576$. Find the coordinate of point 'S'.  	2
	A $(0,-10)$ B $(-10,0)$ C $(0,10)$ (D) $(10,0)$ **Answer:**	

	$16x^2 - 9y^2 = 576$ $\frac{x^2}{36} - \frac{y^2}{64} = 1$ Here $a = 6$ and $b = 8$ $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{64}{36} = \frac{100}{36} \quad \therefore e = \frac{10}{6} = \frac{5}{3}$ $\therefore$ Coordinate of point S is $(ae, 0) = \left(6 \times \frac{5}{3}, 0\right) = (10, 0)$ D. $(10,0)$	
xv.	Find the correlation coefficient using given regression lines.  	2
	A 0.75 B 0.25 C − 0.25 (D) − 0.75 Answer: Let $3x + 4y + 8 = 0$ be y on x — Let $4x + 3y + 7 = 0$ be x on y $y = -\frac{3}{4}x - 2$ — $x = -\frac{3}{4}y - \frac{7}{4}$ $\therefore b_{yx} = -\frac{3}{4}$ — $\therefore b_{xy} = -\frac{3}{4}$ $\therefore r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right)} = \sqrt{\frac{9}{16}} = \pm\frac{3}{4} = \pm 0.75$ Since b_{yx} and b_{xy} are negative, $r = -0.75$ D. -0.75	

SECTION B (70 MARKS)

ANSWER **ANY TEN** QUESTIONS

Question 2

a)	Find the equation of plane passing through the points $(1,-1,2)$ and $(-3,2,-2)$, and perpendicular to plane $x+2y-3z+7=0$.	[4]

Answer:

Any plane passing through the point $(1,-1,2)$ is

$a(x-1)+b(y+1)+c(z-2)=0$ ....(1) ----------- [0.5]

The required plane also passes through point $(-3,2,-2)$

$a(-3-1)+b(2+1)+c(-2-2)=0$

$-4a+3b-4c=0$ ....(2) -----------[0.5]

It is perpendicular to given plane $x+2y-3z+7=0$

$a+2b-3c=0$ ...(3) ----------[0.5]

Solving (2) *and* (3)

$$\frac{a}{\begin{vmatrix}3 & -4\\ 2 & -3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}-4 & -4\\ 1 & -3\end{vmatrix}}=\frac{c}{\begin{vmatrix}-4 & 3\\ 1 & 2\end{vmatrix}}$$ ------- [0.5]

$$\frac{a}{-1}=\frac{-b}{16}=\frac{c}{-11}$$ ------- [0.5]

$a=1,\ b=16$ and $c=11$ --------- [0.5]

Substituting the values of a, b and c in (1)

$1(x-1)+16(y+1)+11(z-2)=0$ ------ [0.5]

$x-1+16y+16+11z-22=0$

$x+16y+11z-7=0$ ------ [0.5]

b)	B-mobile numbers have two series: 17XXXXXX and 16XXXXXX. If the third digit can be any number from 5 to 9, how many unique mobile numbers can be generated?	[3]

Answer:

Each B-mobile number consists of 8 digits, and the first two digits are fixed as 17 or 16.
The third digit can be any of 5, 6, 7, 8, or 9 (5 options).
The remaining 5 digits can be any digit from 0 to 9 (10 options each).

First digit = ${}^{1}p_1 = 1$ ways ----- [0.5]

Second digit = ${}^{2}p_1 = 2$ ways ------ [0.5]

Third digit = ${}^{5}p_1 = 5$ ways ------ [0.5]

Remaining digits = ${}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1$ ------ [0.5]

Total numbers of numbers generated = ${}^{1}p_1 \times {}^{2}p_1 \times {}^{5}p_1 \times {}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1 \times {}^{10}p_1$ ---- [0.5]

$= 1000,000$ unique numbers are generated. ---- [0.5]

Question 3

a) The ranks of 5 students in 3 subjects are given in the table. Use rank correlation to determine the best subject combination with Mathematics. [4]

Students	Mathematics	Chemistry	Physics
A	2	5	2
B	4	1	3
C	5	2	5
D	1	3	4
E	3	4	1

Answer:

Rank in math	Rank in Chemistry	Rank in Physics	d_{mc}	$(d_{mc})^2$	d_{mp}	$(d_{mp})^2$
2	5	2	-3	9	0	0
4	1	3	3	9	1	1
5	2	5	3	9	0	0
1	3	4	-2	4	-3	9
3	4	1	-1	1	2	4
				$\sum(d_{mc})^2 = 32$		$\sum(d_{mp})^2 = 14$

[1 Mark] [1 Mark]

Correlation between Maths and Chemistry $r_{mc} = 1 - \frac{6\sum d_{mc}^2}{n(n^2-1)}$ $r_{mc} = 1 - \frac{6(32)}{5(5^2-1)} = 1 - \frac{192}{120} = -0.6$ ------ [1] Correlation between Maths and Physics $r_{mp} = 1 - \frac{6\sum d_{mc}^2}{n(n^2-1)}$ $r_{mp} = 1 - \frac{6(14)}{5(5^2-1)} = 1 - \frac{84}{120} = 0.3$ ------- [0.5] Mathematics and Chemistry have negative correlation, whereas Mathematics and Physics has positive correlation. Therefore, Physics makes best combination with Mathematics. -----[0.5]	
b) Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(1-2x^2\right)$.	[3]
Answer: $y = \cos^{-1}\left(1-2x^2\right)$. Let $x = \sin\theta$ ----- [0.5] $\theta = \sin^{-1} x$ $y = \cos^{-1}\left(1-2\sin^2\theta\right)$ $y = \cos^{-1}\left(\cos 2\theta\right)$ Q $\cos 2\theta = 1 - 2\sin^2\theta$ ------ [1] $y = 2\theta$ $y = 2\sin^{-1} x$ ----- [0.5] $\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ ----- [1]	
Question 4	
a) Evaluate: $\int \frac{2x+3}{(x-1)\left(x^2+1\right)}\,dx$	[4]
Answer:	

$$\int \frac{2x+3}{(x-1)(x^2+1)}$$

Let $\frac{2x+3}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$ ----- [0.5]

Multiply both sides by the common denominator $(x-1)(x^2+1)$

$2x+3 = A(x^2+1) + (Bx+C)(x-1)$ ----- [0.5]

Now, equate the coefficients of corresponding powers of x

$A+B=0$

$C-B=2$

$A-C=3$

Solving these equations,we get $A=\frac{5}{2}, B=\frac{-5}{2},\ C=\frac{-1}{2}$ ----- $[0.5+0.5+0.5]$

$$\frac{2x+3}{(x-1)(x^2+1)} = \frac{5}{2(x-1)} - \frac{5x}{2(x^2+1)} - \frac{1}{2(x^2+1)}$$

$$\int \frac{2x+3}{(x-1)(x^2+1)}\ dx = \int \frac{5}{2(x-1)} - \int \frac{5x}{2(x^2+1)} - \int \frac{1}{2(x^2+1)} dx$$

$$= \frac{5}{2}\int \frac{1}{(x-1)} - \frac{5}{2}\int \frac{x}{(x^2+1)} - \frac{1}{2}\int \frac{1}{(x^2+1)} dx$$

$$= \frac{5}{2}\log|x-1| - \frac{5}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}(x) + C$$ ---$[0.5+0.5+0.5]$

b) Solve: $\tan\left(\cos^{-1} x\right) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$ [3]

Answer:

$\tan\left(\cos^{-1} x\right) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$

$\tan\left(\cos^{-1} x\right) = \sin\left[\tan^{-1}(2)\right]$ ----- [0.5]

$\tan\left(\cos^{-1} x\right) = \sin\left[\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right]$ ----- [0.5]

$\tan\left(\cos^{-1} x\right) = \frac{2}{\sqrt{5}}$ ----- [0.5]

$\cos^{-1} x = \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$ ----- [0.5]

$x = \cos\left[\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ ----- [0.5]

$x = \frac{\sqrt{5}}{3}$ ----- [0.5]

Question 5	
a) Kinley invests in three types of stock markets: RICB, BNBL and STCB. He invests a total of Nu 50,000. The expected annual returns are 8% for RICB, 6% for BNBL, and 10% for STCB, yielding a total return of Nu 4,200 annually. If his investment in RICB is 3 times the investment in BNBL, use matrix to determine which stock yield maximum dividend?	[4]

Answer: let x, y and z be amount invested in RICB, BNBL and STCB respectively. Then

$x+y+z=50000, \quad 0.08x+0.06y+0.1z=4200, \quad x-3y=0$ ----- [0.5]

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 0.08 & 0.06 & 0.1 \\ 1 & -3 & 0 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 50,000 \\ 4200 \\ 0 \end{bmatrix}}$$ ----- [0.5]

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0.08 & 0.06 & 0.1 \\ 1 & -3 & 0 \end{vmatrix} = 0.3+0.1-0.3=0.1$$ ----- [0.5]

$$adjA = \begin{bmatrix} 0.3 & -3 & 0.04 \\ 0.1 & -1 & -0.02 \\ -0.3 & 4 & -0.02 \end{bmatrix}$$ ----- [1.5]

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{0.1} \begin{bmatrix} 0.3 & -3 & 0.04 \\ 0.1 & -1 & -0.02 \\ -0.3 & 4 & -0.02 \end{bmatrix}$$

$Now\ AX = B \quad \Rightarrow X = A^{-1}B$

$$X = \frac{1}{0.1} \begin{bmatrix} 0.3 & -3 & 0.04 \\ 0.1 & -1 & -0.02 \\ -0.3 & 4 & -0.02 \end{bmatrix} \begin{bmatrix} 50,000 \\ 4200 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24000 \\ 8000 \\ 18000 \end{bmatrix}$$ ----- [0.5]

Dividends yield from different stocks are

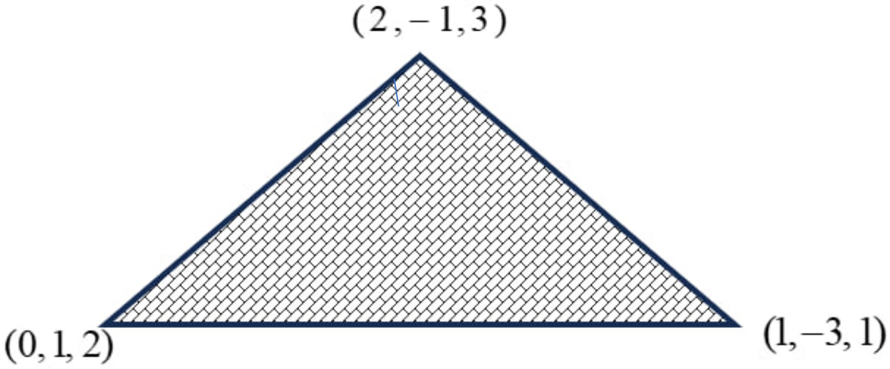
RICB $\Rightarrow 0.08 \times 24000 =$ Nu 1920

BNBL $\Rightarrow 0.06 \times 8000 =$ Nu 480

STCB $\Rightarrow 0.1 \times 18000 =$ Nu 1800

RICB gave maximum yield ----- [0.5]

Therefore, Kinley invests Nu. 24,000, Nu 8000 and Nu 18,000 in RICB, BNBL and STCB respectively.

b)	Find the cost of painting the right triangular roof if the cost of painting 1 square unit is Nu 120. 	[3]
Answer:	Vertices of triangles are; $A(2,-1,3),\ B(0,1,2)\ and\ C(1,-3,1)$ $AB=\sqrt{(2-0)^2+(-1-1)^2+(3-2)^2}=\sqrt{9}=3$ ----- [0.5] $BC=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{18}$ ----- [0.5] $AC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{9}=3$ ----- [0.5] Area $=\frac{1}{2}bh=\frac{1}{2}\times 3\times 3=4.5\,\text{unit}^2$ ----- [1] $\therefore$ Total cost $=4.5\times 120=$ Nu 540 ----- [0.5]	
Question 6		
a)	A car travels along the straight road. Its distance d in kilometers in time t is given by $d=40t+2t^2$. The fuel consumption rate c in liters per hour is given by $c=0.1t^2+0.8t+2$.	[4]
i)	Calculate the speed of the car in time $t=3$ hours. **Answer:** $d(t)=40t+2t^2$ Rate of change in distance w.r.t time t is; $\frac{dd}{dt}=40+4t$ −−−−−−−−−−[0.5] When $t=3$ hours $\frac{dd}{dt}=40+4\times 3=52$ −−−−−−−−−−[0.5] Speed of the car is 52 km/hour −−−−−−−−−−[0.5]	[1.5]

ii) What is the rate of change in distance with respect to fuel consumption in 2 hours?	[2.5]

Answer:

Rate of change in distance w.r.t fuel consumption is $\frac{dd}{dc}$; −−−−−[0.5]

$d = 40t + 2t^2$

$\frac{dd}{dt} = 40 + 4t$

$c = 0.1t^2 + 0.8t + 2$

$\frac{dc}{dt} = 0.2t + 0.8$ −−−−−−−−−−−[0.5]

$\frac{d(d)}{d(c)} = \frac{dd}{dt} \times \frac{dt}{dc} = \frac{40+4t}{0.2t+0.8}$ −−−−−−−−−−−[1]

$\frac{dd}{dc}_{t=2} = \frac{40+4\times 2}{0.2\times 2+0.8} = 40$

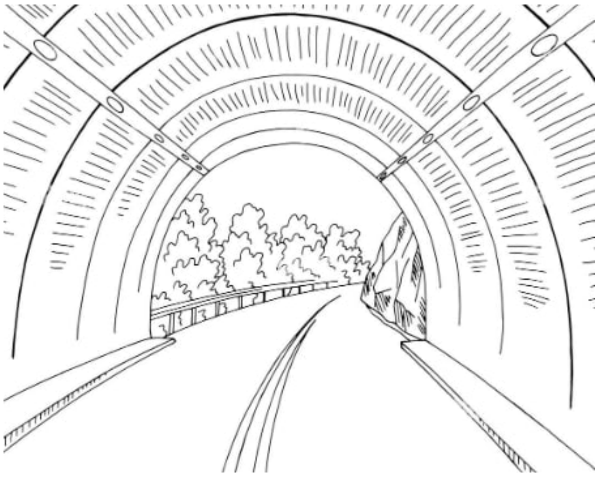
Rate of change in distance w.r.t fuel consumption is $\frac{dd}{dc} = 40km/litre$ −−−−−−−−−−−[0.5]

b) Match the following: [3]

Column A	Column B
i) Independent event	a) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$
ii) Complementary event	b) $P(A/B) = \frac{P(A \cap B)}{P(B)}$
iii) Conditional event	c) $P(A \cap B) = P(A) \times P(B)$
	d) $P(A) + P(B) = 1$
	e) $P(\overline{A}) = 1 - P(A)$

Answer:

Column A	Column B	Marks	Mar
i) Independent	c) $P(A \cap B) = P(A) \times P(B)$	[1]	
ii) Complementary	e) $P(\overline{A}) = 1 - P(A)$	[1]	
iii) Conditional	b) $P(A/B) = \frac{P(A \cap B)}{P(B)}$	[1]	

Question 7	
a) The given tunnel can be modeled by an equation $25x^2 + 16y^2 = 400$. Decide a dimension of vehicle that can safely pass through this tunnel assuming that the vehicle drives through the centre of tunnel road. 	[4]
Answer: $25x^2 + 16y^2 = 400.$ $\frac{x^2}{16} + \frac{y^2}{25} = 1,$ which gives $a = 5$ and $b = 4.$ ----- [0.5] $\therefore$ width of tunnel is $= 2b = 2 \times 4 = 8$ units. ----- [0.5] Height is $a = 5$ units ----- [0.5] Truck can have any width not exceeding 8 units and height not exceeding 5 units. Sample response: let the width of truck be 4 units $(x = 2)$ ----- [1] $25(2)^2 + 16y^2 = 400$ $16y^2 = 400 - 100$ $y^2 = \frac{300}{16}$ ----- [1] $y = \sqrt{\frac{300}{16}} = 4.3$ The truck can have width of 4 units and height 4.3 units to safely pass through the tunnel. ----- [0.5]	

b) Show that $\beta(Adj\beta)+(Adj\beta)\beta=2(\lvert\beta\rvert I)$ if $\beta=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. **Answer:** $\beta=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $Adj\beta=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\beta(Adj\beta)=\begin{bmatrix} ad-bc & -ab+ab \\ cd-cd & -bc+ad \end{bmatrix}=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$-------[0.5] $(Adj\beta)\beta=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$ considering commutative property −[0.5] $\lvert\beta\rvert=(ad-bc)$----------------------------------[0.5] $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$----------------------------------[0.5] $\lvert\beta\rvert I=(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$--------------[0.5] Showing $\beta(Adj\beta)+(Adj\beta)\beta=2(\lvert\beta\rvert I)$ $\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}+\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}=2\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$ --[0.5] $\begin{bmatrix} 2(ad-bc) & 0 \\ 0 & 2(ad-bc) \end{bmatrix}=\begin{bmatrix} 2(ad-bc) & 0 \\ 0 & 2(ad-bc) \end{bmatrix}$		[3]
Question 8		
a) Find the limits of the integration for the curve $4x^2+5y^2=100$ and the volume of the solid formed by one complete rotation about x −axis. **Answer:** To find the limits, find the vertices of ellipse (x −coordinates) $4x^2+5y^2=100$ $\frac{x^2}{25}+\frac{y^2}{20}=1, \quad a=\pm 5$ $\therefore$ The limits are $x=-5$ and $x=5$. ------[1]		[4]

$V=\pi\int_a^b y^2dx=\pi\int_{-5}^{5}\left(20-\frac{4}{5}x^2\right)dx$ -----[0.5] $=2\pi\int_0^5\left(20-\frac{4}{5}x^2\right)dx$ -----[0.5] $=2\pi\left[20x-\frac{4}{5}\frac{x^3}{3}\right]_0^5$ -------[1] $=2\pi\left[20\times5-\frac{4(5)^3}{15}\right]$ ------- [0.5] $=2\pi\left[100-\frac{500}{15}\right]$ $=2\pi\left[\frac{200}{3}\right]$ $=\frac{400\pi}{3}$ unit^3 --------[0.5]	
b) Find the distance between the parallel planes $4x+3y-12z-4=0$ and $4x+3y-12z+6=0$. **Answer:** x-intercept of the plane $4x+3y-12z-4=0$ is $(1,0,0)$ .... [1] The perpendicular distance of point $(1,0,0)$ from the plane $4x+3y-12z+6=0$ is $d=\frac{4(1)+3(0)-12(0)+6}{\sqrt{4^2+3^2+(-12)^2}}$ ....[1] $=\frac{10}{\sqrt{169}}$ $=\frac{10}{13}$ Units ....[1]	[3]
Question 9	
a) Find the general solution of $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$.	[4]

	We have $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$ It is of the form $\frac{dy}{dx}+Py=Q$ Where $P=\tan x$ and $Q=2x+x^2\tan x$ ------------[0.5] $I.F=e^{\int p.dx}=e^{\int \tan x.dx}=e^{\log\sec x}=\sec x$ ------------[0.5] The general solution is $y(I.F)=\int Q(I.F)dx+c$ --------------------[0.5] $ysecx=\int\left(2x+x^2\tan x\right)(\sec x)dx+c$ $ysecx=\int 2x\sec xdx+\int x^2\tan x\sec xdx+c$ ----------[0.5] $ysecx=\int 2x\sec xdx+x^2\sec x-\int 2x\sec xdx+c$ --------[1] $ysecx=x^2\sec x+c$ ----------------------[0.5] $y=x^2+c\cos x$ ------------------------[0.5]	
b)	Express the complex number $z=5\sqrt{3}-5i$ into polar form. **Answer:** $a=5\sqrt{3}$ and $b=-5$ $r=\lvert z\rvert=\sqrt{\left(5\sqrt{3}\right)^2+(-5)^2}=\sqrt{75+25}=10$ --- [1] $\alpha=\tan^{-1}\left\lvert\frac{b}{a}\right\rvert=\tan^{-1}\left\lvert\frac{-5}{5\sqrt{3}}\right\rvert=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$ --- [1] Since the z lies in the 4^{th} quadrant, $\theta=-\alpha=-\frac{\pi}{6}$ --- [0.5] $\therefore z=10\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]$ --- [0.5]	[3]
Question 10		
a)	Check whether the equation $4x^2+4xy+y^2-6x-3y-4=0$ represents a pair of straight lines. Find the equations. **Answers:** $4x^2+4xy+y^2-6x-3y-4=0$ $a=4,\ h=2,\ b=1,\ g=-3,\ f=-\frac{3}{2},\ c=-4$ ------ [0.5] * $\Delta=acb+2fgh-af^2-bg^2-ch^2=0$ $(4)(1)(-4)+2\left(-\frac{3}{2}\right)(-3)(2)-4\left(-\frac{3}{2}\right)^2-1(-3)^2-(-4)(2)^2=0$ $-16+18-9-9+16=0$ $0=0$ ----- [1] * $h^2\geq ab,\quad g^2\geq ac,\quad f^2\geq bc$ ------[0.5] $2^2\geq(4)(1),\quad(-3)^2\geq 4(-4),\quad\left(-\frac{3}{2}\right)^2\geq 1(-4)$ $4\geq 4,\quad 9\geq -16,\quad \frac{9}{4}\geq -4$ All true ----- [1] Since it satisfies both the conditions, the given equation represents pairs of straight lines.	[5]

$4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

$4x^2 + (4y-6)x + y^2 - 3y - 4 = 0$

$a = 4, \quad b = 4y - 6, \quad c = y^2 - 3y - 4$ ----- [0.5]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4y-6) \pm \sqrt{(4y-6)^2 - 4(4)(y^2 - 3y - 4)}}{2(4)}$$

$8x = 6 - 4y \pm \sqrt{16y^2 - 48y + 36 - 16y^2 + 48y + 64}$

$8x = 6 - 4y \pm \sqrt{100}$ ------ [1]

$8x = 6 - 4y + 10$ or $8x = 6 - 4y - 10$

$8x + 4y - 16 = 0$ or $8x + 4y + 4 = 0$

$\therefore$ Lines are $2x + y - 4 = 0$ *and* $2x + y + 1 = 0$ ----- [0.5]

b) Examine the principal values of the following functions. [2]

Function	Value	Principal Value
i) $\sin^{-1} x$	$\frac{2\pi}{3}$	-----------
ii) $\cos^{-1} x$	$\frac{2\pi}{3}$	-----------
iii) $\tan^{-1} x$	$\frac{2\pi}{3}$	-----------
iv) $\cot^{-1} x$	$\frac{2\pi}{3}$	-----------

Answer:

Function	Value	Principal Value	Marks
i) $\sin^{-1} x$	$\frac{2\pi}{3}$	$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$	[0.5]
ii) $\cos^{-1} x$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	[0.5]
iii) $\tan^{-1} x$	$\frac{2\pi}{3}$	$\frac{2\pi}{3} - \pi = -\frac{\pi}{3}$	[0.5]
iv) $\cot^{-1} x$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	[0.5]

Question 11		
a)	An Engineer is designing a square-based open tank to address water shortages. The tank is designed to hold V cubic units of water. If you are an Engineer, decide the height of the tank to ensure cost-effectiveness.	[4]

Answer:

Let x be the sides of square based and h be the height of water tank.
Volume $= x^2 h$
$h = \frac{V}{x^2}$ ——[0.5]

Let S be the surface area of the tank, then

$S = x^2 + 4xh$ ———————[0.5]

$S = x^2 + 4x\left(\frac{V}{x^2}\right) = x^2 + \frac{4V}{x}$
$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$ ———————[0.5]

For maximum and minimum $2x - \frac{4V}{x^2} = 0$ ———————[0.5]

$2x^3 = 4V$

$x^3 = 2V$

$x = \sqrt[3]{2V}$ ———————[0.5]

Also $\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$,

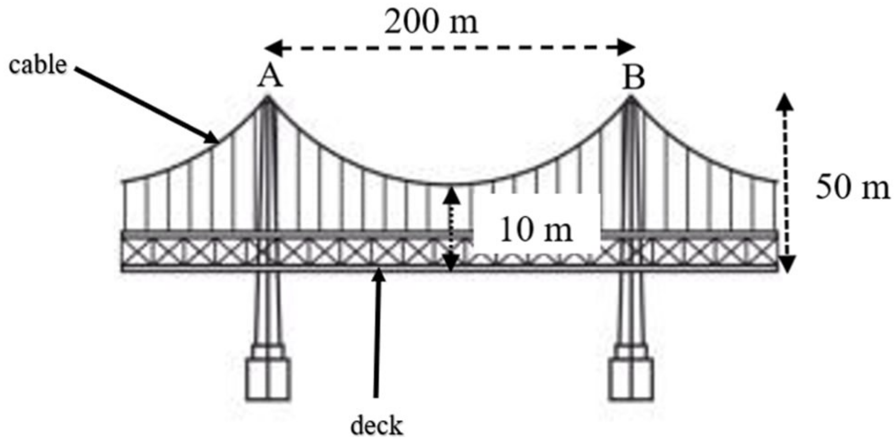
When $x = \sqrt[3]{2V}; \frac{d^2S}{dx^2} > 0$ ———————[0.5]

Surface area is minimum when $x = \sqrt[3]{2V}$

$\therefore height = \frac{V}{x^2} = \frac{V}{\frac{2V}{x}} = \frac{x}{2}$ units ———————[0.5]

The tank is cost effective when height of the tank is half the the side length. ———————[0.5]

b)	Find the equation of an arc formed by the suspension bridge cable between towers A and B, assuming a vertical axis through the center of the towers and a horizontal axis through the base of the deck.	[3]



Answer:

The equation of parabola with vertex(h,k) formed by a cable is

$(x-h)^2 = 4a(y-k)$ ----[0.5]

We have $(h,k) = (0,10)$

$x^2 = 4a(y-10)$........................(i) -----[0.5]

Considering the parabola passes through point B(100,50) -----[0.5]

$100^2 = 4a(50-10)$

$10,000 = 160a$

$a = \frac{10,000}{160} = \frac{10,000}{160} = \frac{125}{2}$ ------[0.5]

Substituting $a = \frac{125}{2}$ in equation (i)

$x^2 = 4 \times \frac{125}{2}(y-10)$ --------[0.5]

$x^2 = 250(y-10)$

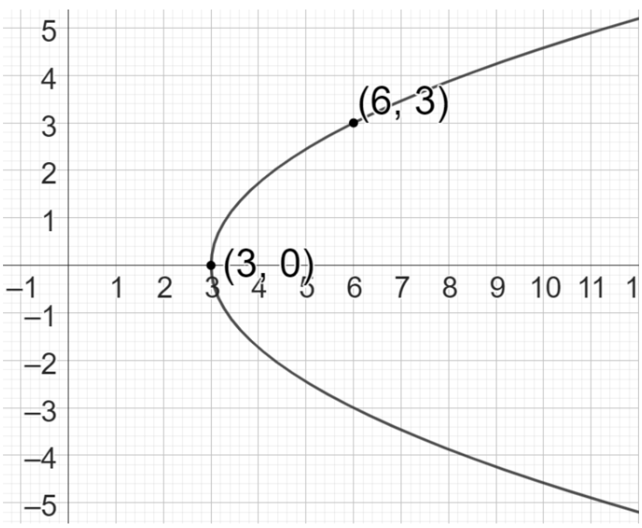
$x^2 = 250y - 2500$ -------[0.5]

Question 12

a) Yethro and Damchoe appear for an interview for two vacancies. They will be selected if they score at least 60% in each criterion. [4]

Criteria	Viva [10]	Academic [10]	Leadership [5]	Volunteerism [5]	Presentation [5]	Total
Yethro	4	5	4	5	3	21
Damchoe	5	6	2	2	1	16

What is the probability that

i. only one of them will be selected? **Answer:** Let probability of Yethro being selected be $P(A)$ and Damchoe being selected be $P(B)$ From the score sheet we know that, $P(A)=\frac{3}{5}$ ------------[0.5] $P(B)=\frac{1}{5}$ ------------[0.5] $P(\overline{A})=1-\frac{3}{5}=\frac{2}{5}$ ------[0.5] $P(\overline{B})=1-\frac{1}{5}=\frac{4}{5}$ ------[0.5] P(one of them will be selected) $=P(A)P(\overline{B})+P(B)P(\overline{A})$ ------[0.5] P(one of them will be selected) $=\frac{3}{5}\times\frac{4}{5}+\frac{1}{5}\times\frac{2}{5}=\frac{14}{25}$ ------[0.5]	[3]
ii. at least one of them will be selected? **Answer:** P(at least one of them will be selected) $=1-P(\overline{A})P(\overline{B})$ ------[0.5] P(at least one of them will be selected) $=1-\frac{2}{5}\times\frac{4}{5}$ $=1-\frac{8}{25}=\frac{17}{25}$ --------[0.5]	[1]
b) Find the area bounded by the curve, the x-axis and $x=10$.  **Answer:**	[3]

Equation of the given parabola whose vertex is not an origin is; $(y-h)^2 = 4a(x-k)$ $(y-0)^2 = 4a(x-3)$ $y^2 = 4a(x-3)$ ---------[0.5] This parabola passes through the point (6,3) $9 = 4a(6-3)$ $a = \frac{3}{4}$ ---------[0.5] $\therefore\ y^2 = 4\times\frac{3}{4}(x-3)$ $y^2 = 3x-9$ $y = (3x-9)^{\frac{1}{2}}$ ---------[0.5] Area of the parabola from $x=3$ to $x=10$ $\int_3^{10}(3x-9)^{\frac{1}{2}} = \frac{(3x-9)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\times\frac{1}{3} = \left[\frac{2(3x-9)^{\frac{3}{2}}}{9}\right]_3^{10}$ --------[0.5] $= \left[\frac{2(3(10)-9)^{\frac{3}{2}}}{9}\right] - \left[\frac{2(3(3)-9)^{\frac{3}{2}}}{9}\right] = \left[\frac{2(21)^{\frac{3}{2}}}{9}\right] - 0$ $= \left[\frac{2(21)^{\frac{3}{2}}}{9}\right] = \left[\frac{2\sqrt{21\times21\times21}}{9}\right] = \frac{2\times\cancel{21}\times\sqrt{21}}{\cancel{9}}$ ------[0.5] $= \frac{14\sqrt{21}}{3}$ square units --------[0.5]	
Question 13	
a) Illustrate and describe the region of Argand's plane represented by the inequality $\left\|\frac{z-3i}{z+3i}\right\| \le \sqrt{2}$. **Answer:** $\left\|\frac{x+iy-3i}{x+iy+3i}\right\| \le \sqrt{2}$ ---- [0.5] $\sqrt{x^2+(y-3)^2} \le \sqrt{2}\sqrt{x^2+(y+3)^2}$ ---- [0.5] Squaring on both sides $x^2+y^2-6y+9 \le 2(x^2+y^2+6y+9)$ ---- [0.5] $x^2+y^2-6y-2x^2-2y^2-12y \le 18-9$ $-x^2-y^2-18y \le 9$ ---- [0.5] $x^2+y^2+18y \ge -9$ ---- [0.5] $x^2+y^2+18y+\left(\frac{18}{2}\right)^2 \ge -9+\left(\frac{18}{2}\right)^2$ ---- [0.5] $x^2+(y+9)^2 \ge 72$ ---- [0.5] $x^2+(y+9)^2 \ge \left(\sqrt{72}\right)^2$ Locus is the region along and outside the circle with centre $(0,-9)$ and radius $\sqrt{72}$ ---- [0.5]	[4]

b) A study is conducted to examine the relationship between the continuous assessment (CA) mark and the mark obtained in a Mathematics exam. The data collected from 5 students are as follows: [3]

Student	A	B	C	D	E
CA	5	10	3	8	6
Exam	60	75	50	70	65

If a student with 7 marks in the CA, predict his score in the exam.

Answer:

CA(x)	Exam(y)	xy	x^2
5	60	300	25
10	75	750	100
3	50	150	9
8	70	560	64
6	65	390	36
$\sum x = 32$	$\sum y = 320$	$\sum xy = 2150$	$\sum x^2 = 234$

0.5 [0.5+0.5]

$$\bar{x} = \frac{\sum x}{n} = \frac{32}{5} = 6.4$$

$$\bar{y} = \frac{\sum y}{n} = \frac{320}{5} = 64$$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{5(2150) - 32 \times 320}{5(234) - (32)^2} = \frac{510}{146} = 3.49 \quad ----[0.5]$$

$\therefore$ Equation y on x is

$y - 64 = 3.49(x - 6.4)$

$y = 3.49x - 22.34 + 64$

$y = 3.49x + 41.66$ ----[0.5]

Probable exam score of the student with CA 7 is

$y = 3.49(7) + 41.66 \approx 66$ ----[0.5]