SECTION A (30 MARKS)

ANSWER **ALL** QUESTIONS

For each question, there are four alternatives A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than one circled, NO score will be awarded. **[30]**

i. The additive inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -4 \end{bmatrix}$ is

A $\begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix}$.

[✓ B] $\begin{bmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-5} & \mathbf{4} \end{bmatrix}$.

C $\frac{1}{2}\begin{bmatrix} -4 & 5 \\ 2 & -3 \end{bmatrix}$.

D $\frac{1}{2}\begin{bmatrix} -4 & -2 \\ -5 & -3 \end{bmatrix}$.

Answer: B $\begin{bmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-5} & \mathbf{4} \end{bmatrix}$

Since the sum of matrix and its inverse is zero matrix, the additive inverse of $\begin{bmatrix} -3 & 2 \\ 5 & -4 \end{bmatrix}$ is $\begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$.

Criteria	**Marks**
Circles the correct option	**2**
Circles more than ONE alternative	**0**
Circles none of the alternatives	**0**

ii. Which of the following is the derivative of $1+\frac{1}{x}$?

A x

B x^2

C $-\frac{1}{x}$

[✓ D] $-\frac{\mathbf{1}}{\boldsymbol{x^2}}$

Answer: D $-\frac{1}{x^2}$

$$\frac{d}{dx}\left(1+\frac{1}{x}\right)=0-\frac{1}{x^2}=-\frac{1}{x^2}$$

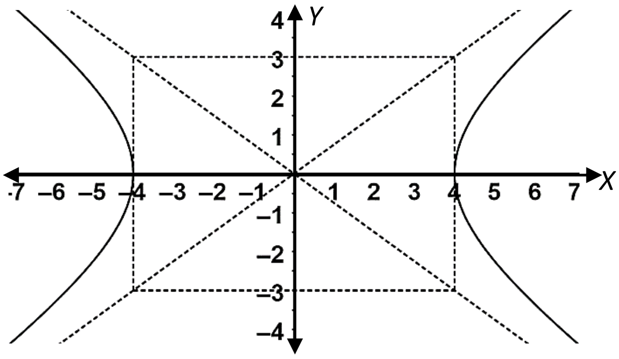
iii. The eccentricity of the given conic section is

A $\frac{7}{16}$.

B $\frac{\sqrt{7}}{4}$.

[✓ C] $\frac{5}{4}$.

D $\frac{25}{16}$.



Answer: C $\frac{5}{4}$.

Since the given conic section is a hyperbola, $a=\pm4,\ b=\pm3$.

We have $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{9}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$.

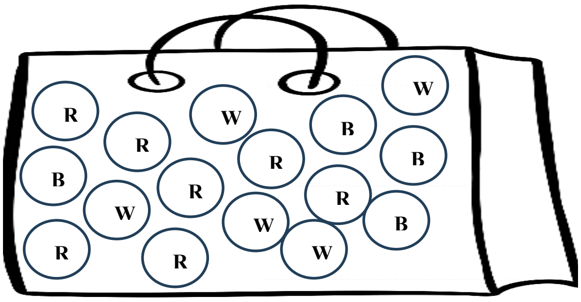
iv. A bag contains identical balls of a different colours. When you draw two balls, one after another randomly without replacement, you will win a prize at each draw if the ball drawn is black. What is the probability of winning at least one prize?

A 0.25

[✓ B] 0.45

C 0.67

D 0.95



Key
R – Red
W – White
B – Black

Answer: B 0.45

The required probability is:

P(winning in 1st) and P(not winning in 2nd)+P(not winning in 1st) and P(winning in 2nd) + P(winning in 1st) and P(winning in 2nd)

$$=\frac{4}{16}\times\frac{12}{15}+\frac{12}{16}\times\frac{4}{15}+\frac{4}{16}\times\frac{3}{15}$$

$$=\frac{9}{20}=0.45$$

OR

Probability that at least one of them winning is $1-$ Probability of both not winning

$$=1-\frac{12}{16}\times\frac{11}{15}$$

$$=\frac{9}{20}=0.45$$

v. The following table depicts the information related to loan products.

Loan Products	**Fixed Rate**	**Floating Rate**	**Max tenure (years)**
Consumer Loan	11.00%	9.50%	5
Personal Loan	15.00%	13.50%	5
Manufacturing & Industry	12.75%	11.50%	20

An entrepreneur has taken the “Manufacturing & Industry” loan and is paying back Nu 11,538.12 per month. If the loan was availed at the fixed interest rate, what was the sum borrowed?

A Nu 12,770,242

B Nu 12,635,984

C Nu 1,085,940

[✓ D] Nu 1,000,000

Answer: D Nu 1,000,000

Give: $a=11,538.12,\quad i=\frac{12.75}{12\times100}=0.010625,\ n=12\times20=240$

$$P=\frac{a}{i}\left(1-(1+i)^{-n}\right)$$

$$P=\frac{11,538.12}{0.010625}\left(1-1.010625^{-240}\right)=1,000,000$$

vi. A store owner purchased a washing machine for Nu 5,000. In order to attract customers, the owner offered a discount of 20% on the marked price. However, the owner wants to make a profit of 50% on the sale. What price should he mark on the washing machine to achieve this goal?

[✓ A] Nu 9,375

B Nu 7,500

C Nu 6,000

D Nu 2,000

Answer: A Nu 9,375

Since the net profit is 50% of 5000, the sale price should be 150% of $5000=1.5\times5000=7500$.

Since the sale price is after giving 20% discount from the marked price, 80% of Marked Price $= 7500$

$\Rightarrow 0.8(\text{Marked Price}) = 7500$

$\therefore$ Marked Price $= \dfrac{7500}{0.8} = 9375.$

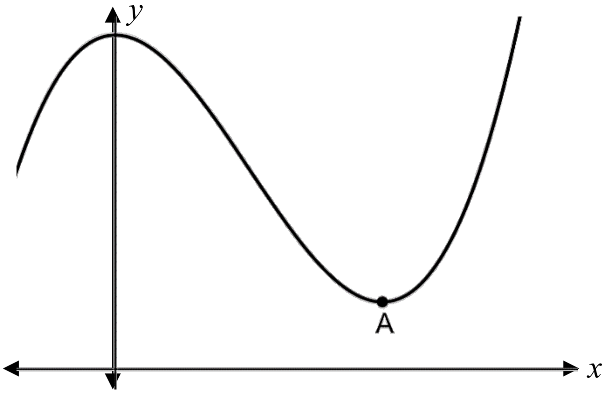
vii. The equation of the given curve is $y = x^3 - 3x^2 + 5$. Find the coordinates of the point **A**.

[✓ A] (2, 1)

B (3, 1)

C (4, 2)

D (5, 2)



Answer: A (2, 1)

Given the function: $y = x^3 - 3x^2 + 5$

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$\frac{dy}{dx} = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0 \ \ and \ \ 2$$

$$\frac{d^2y}{dx^2} = 6x - 6. \ \text{At } x = 2, \frac{d^2y}{dx^2} = 6(2) - 6 = 6 > 0$$

$\therefore$ there is minimum point at $x = 2$

The y-coordinate of the point A is $2^3 - 3(2)^2 + 5 = 1$

$\therefore$ The coordinates of point A is (2, 1)

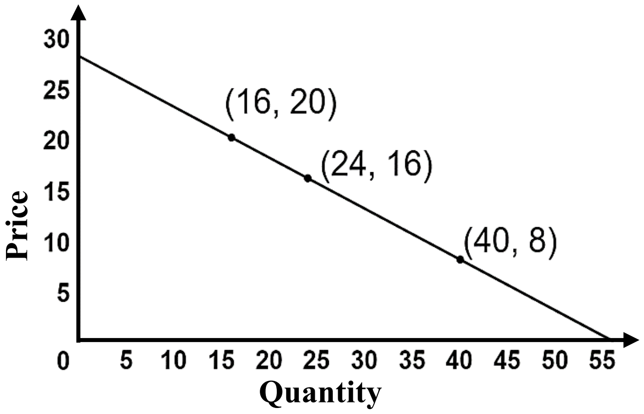
viii. A manufacturing firm aims to analyze the relationship between the demand for their commodity and its corresponding price. They have access to a demand curve which is provided below. The demand function of the commodity is

A $p = 28 - 2x$.

B $p = -8 - 2x$.

[✓ C] $\boldsymbol{p = 28 - 0.5x}$.

D $p = 8 - 0.5x$.



Answer: C $\boldsymbol{p = 28 - 0.5x}$.

Since the demand curve is linear, let the demand function be $p = a + bx$. Substituting (24,16), we get $16 = a + 24b \Rightarrow a = 16 - 24b$

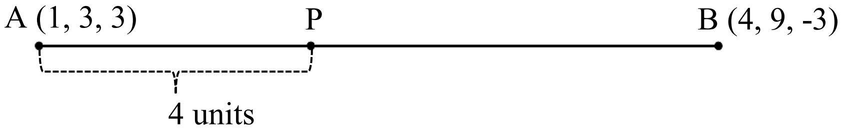
Substituting (40, 8), we get $8 = a + 40x \Rightarrow 8 = 16 - 24b + 40b$

$$16b = -8 \Rightarrow b = -0.5$$

$$a = 16 - 24(-0.5) = 28$$

$\therefore$ Demand funtion is: $p = 28 - 0.5x$

ix. Find the distance between the points P and B in the following diagram.



A 9 units

[✓ B] 5 units

C 4 units

D 2 units

Answer: B 5 units

Distance between A and B: $D = \sqrt{(4-1)^2 + (9-3)^2 + (-3-3)^2} = 9$

Since the distance between A and P is 4 units, the distance between P and B will be $9 - 4 = 5$ units.

x. Two regression lines of a certain data were found to be $7x - 11y + 6 = 0$ and $2x - 3y + 1 = 0$. Evaluate the value of y for $x = 18$.

A 12.5

B 12.4

C 12.3

[✓ D] 12.0

Answer: D 12.0

Since we have to find the value of y using the given value of x, we need to identify the regression equation of 'y on x' from the two given equations.

Let $7x-11y+6=0$ be the equation of 'y on x' and $2x-3y+1=0$ be the equation of "x on y'. Then

$$-11y=-7x-6 \Rightarrow y=\frac{7}{11}x+\frac{6}{11} \Rightarrow b_{yx}=\frac{7}{11}$$

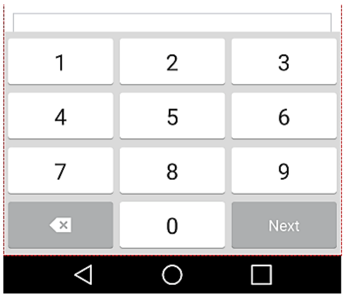
$$2x=3y-1 \Rightarrow x=\frac{3}{2}y-\frac{1}{2} \Rightarrow b_{xy}=\frac{3}{2}$$

$$r=\sqrt{\frac{7}{11}\times\frac{3}{2}}=0.977<1. \quad \therefore \text{Our assumption is true}$$

$\therefore$ The value of y when $x=18$ is

$$y=\frac{7}{11}(18)+\frac{6}{11}=12.$$

xi. From the digits shown in the phone keypad, find the number of four-digit passwords that can be formed without digits being repeated.



[✓ A] 5040

B 4536

C 3024

D 2688

Answer: A 5040

Since digits are not repeated and also 0 can be the leading digit for pincodes, the number of pincodes that can be formed is ${}^{10}P_4=5040$ or $10\times9\times8\times7=5040$.

xii. Find the anti-derivative of $\sqrt{x}-\frac{1}{\sqrt{2x}}$.

A $\frac{1}{2\sqrt{x}}+\frac{1}{\sqrt{8}x^{3/2}}+c$

B $\frac{3}{2}x^{3/2}-2\sqrt{x}+c$

[✓ C] $\boldsymbol{\frac{2}{3}x^{3/2}-\sqrt{2x}+c}$

D $\quad \dfrac{1}{\sqrt{2x}}+\dfrac{1}{2\sqrt{2}x^{3/2}}+c$

Answer: C $\boldsymbol{\dfrac{2}{3}x^{3/2}-\sqrt{2x}+c}$

$$\int \sqrt{x}-\frac{1}{\sqrt{2x}}\,dx=\int\left(x^{1/2}-\frac{1}{\sqrt{2}}x^{-1/2}\right)dx=\frac{x^{3/2}}{\frac{3}{2}}-\frac{x^{1/2}}{\sqrt{2}\times\frac{1}{2}}+c$$

$$=\frac{2}{3}x^{3/2}-\frac{2}{\sqrt{2}}x^{1/2}+c$$

$$=\frac{2}{3}x^{3/2}-\frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}\times\sqrt{x}+c$$

$$=\frac{2}{3}x^{3/2}-\sqrt{2x}+c$$

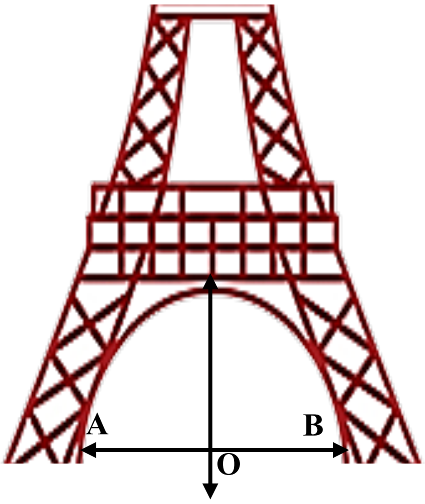
xiii. Two engineering students have designed a model of a tower as shown below. The arch of the model was drawn using the equation $8x^2+2y^2=800$. Find the width of the arch from points A to B.

A 10 units

[✓ B] 20 units

C 30 units

D 40 units



Answer: B 20 units

Given the equation of the ellipse: $8x^2+2y^2=800$

$$\frac{8x^2+2y^2=800}{800}=\frac{x^2}{100}+\frac{y^2}{400}=1$$

$\Rightarrow a=20,\ b=10$

$\therefore$ The distance between A and B is $2b=20$ units.

xiv. The corelation coefficient between the marks of Accountancy and Business Mathematics is $\frac{2}{3}$. If the sum of the squares of the differences in ranks is 55, how many marks of each subject were taken to find the correlation?

A 12

B 11

[✓ C] 10

D 9

Answer: C 10

Given $r=\frac{2}{3}$, $\sum D^2=55$. Using the formula $r=1-\frac{6\sum D^2}{n(n^2-1)}$,

$$1-\frac{6(55)}{n(n^2-1)}=\frac{2}{3}$$

$$3n^3-3n-990=2n^3-2n$$

$$n^3-n-990=0$$

$$n(n^2-1)=990$$

$$n(n-1)(n+1)=990$$

$$(n-1)(n)(n+1)=9(10)(11)$$

By comparing, $n=10$

xv. A company decides to save Nu 200,000 from its profit at the end of each year in a bank which pays compound interest at 6% p.a. The total savings accumulated at the end of 10 years is

A Nu 58,107,290.

B Nu 36,317,050.

C Nu 2,794,330.

[✓ D] Nu 2,636,160.

Answer: D Nu 2,636,160

Given: $a=200,000,\ i=0.06,\ n=10$

$$A=\frac{a}{i}\left((1+i)^n-1\right)$$

$$A=\frac{200000}{0.06}\left(1.06^{10}-1\right)$$ ß

$$A\approx 2,636,160$$

Therefore, the total amount saved is Nu 2,63,160

SECTION B (70 MARKS)

ANSWER ANY **TEN** QUESTIONS

Question 2

a) Find a 2×2 matrix *P* such that $P\begin{bmatrix}3 & -4\\ 2 & 1\end{bmatrix}=\begin{bmatrix}11 & 0\\ 0 & 11\end{bmatrix}$. **[3]**

Answer:

Let $Q=\begin{bmatrix}3 & -4\\ 2 & 1\end{bmatrix}$

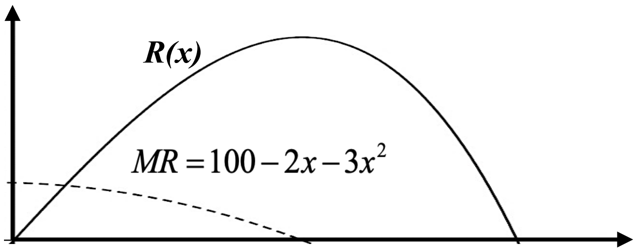
Since $P\begin{bmatrix}3 & -4\\ 2 & 1\end{bmatrix}=\begin{bmatrix}11 & 0\\ 0 & 11\end{bmatrix}, P=\begin{bmatrix}11 & 0\\ 0 & 11\end{bmatrix}\begin{bmatrix}3 & -4\\ 2 & 1\end{bmatrix}^{-1}$ *,Thus we have to find* Q^{-1}.......(0.5)

$|Q|=3+8=11$..(0.5)

$$\left.\begin{aligned}Q^{-1}&=\frac{1}{|Q|}\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}\\ Q^{-1}&=\frac{1}{11}\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}\end{aligned}\right\}$$ ....(1)

$\therefore P=\begin{bmatrix}11 & 0\\ 0 & 11\end{bmatrix}\frac{1}{11}\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}=\frac{1}{11}\begin{bmatrix}11 & 44\\ -22 & 33\end{bmatrix}=\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}$..(1)

b) In the given graph, the dotted curve represents the marginal revenue function, and the solid curve represents the total revenue function. **[4]**



i. Determine the equation of the solid curve. **[2]**

Answer:

$R(x)=\int MRdx=\int\left(100-2x-3x^2\right)dx=100x-x^2-x^3+c$......................(1)

$$\left.\begin{aligned}&R(0)=100(0)-0^2-0^3+c=0\Rightarrow c=0\\ &\therefore R(x)=100x-x^2-x^3\end{aligned}\right\}$$ ....(1)

ii. Determine the demand function and the price when $x=5$. **[2]**

Answer:

Demand function: $p = \frac{R(x)}{x} = \frac{100x - x^2 - x^3}{x} = 100 - x - x^2$.....................(1)

$\therefore$ The price demand for $x = 5$ is $100 - 5 - 5^2 = 70$......................................(1)

Question 3

a) *As part of the Druk Gyalpo's Relief Kidu to address the challenges posed by the COVID-19 pandemic, loans approved prior to April 2020 were provided with deferred payments and complete interest waiver. Loans approved after April 2020 had the option of deferring payments, but without an interest waiver.*

Dawa obtained a loan on 26th May, 2020, at an interest rate of 9% p.a., and decided to defer the loan for one year. The monthly instalment was Nu 5,000.

i. Find the total amount to be paid at the end of one year to bring the loan up to date. **[1.5]**

Answer: Since the loan was obtained after April 2020, there is not interest waiver. Also since the loan was deferred for one year, Dawa has missed 12 instalments. Thus, Dawa must pay the future amount of Nu 5000 for 12 periods:

$a = 5000,\ i = \frac{9}{12 \times 100} = 0.0075,\ n = 12$..(0.5)

$$\left.\begin{array}{l} A = \frac{a}{i}\left((1+i)^n - 1\right) = \frac{5000}{0.0075}\left(1.0075^{12} - 1\right) \approx 62{,}537.93 \\ \therefore \text{Dawa must pay Nu 62,537.93 to bring her loan upto date.} \end{array}\right\}\text{.......................(1)}$$

OR students may calculate by computing each parts through round off, and the answers may defer by hundreds.

Possible answers could be between 62,530 - 62,670

ii. If the tenure of the loan was for 5 years, what was the sum of loan sanctioned? **[1.5]**

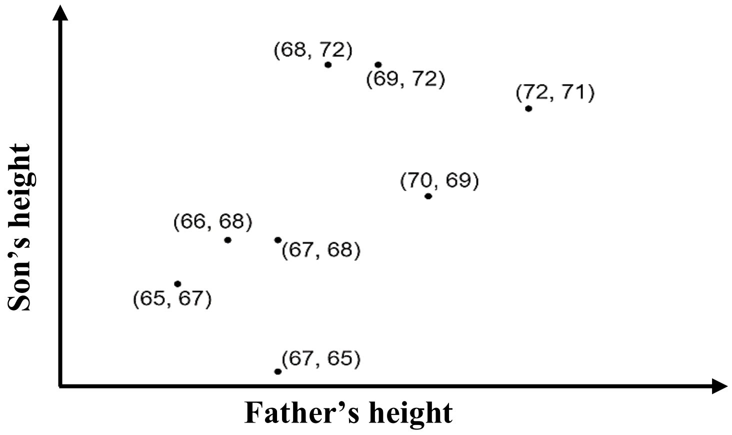
Answer: The sum of the loan obtained is the present value of Nu 5000 monthly at 9% p.a. of five years.

$a = 5000, i = 0.0075,\ n = 12 \times 5 = 60$

$P = \frac{a}{i}\left(1 - (1+i)^{-n}\right)$..(0.5)

$$\left.\begin{array}{l} = \frac{5000}{0.0075}(1 - 1.0075^{-60}) = 240{,}866.87 \\ \therefore \text{ The sum of loan obtained is Nu 240,866.87} \end{array}\right\}\text{..(1)}$$

b) The heights of fathers and their sons were measured in inches, and represented in the scatter plot shown below. Find rank correlation coefficient between the heights of the fathers and sons. **[4]**



Answer:

x	y	R_x	R_y	$D = R_x - R_y$	D^2
65	67	8	7	1	1
66	68	7	5.5	1.5	1.25
67	65	5.5	8	-2.5	6.25
67	68	5.5	5.5	0	0
68	72	4	1.5	2.5	6.25
69	72	3	1.5	1.5	2.25
70	69	2	4	-2	4
72	71	1	3	-2	4
				$\sum D^2 = \mathbf{26}$	

……………………………………………………………………..…….(1.5)

$$\text{Correction factors for } x\text{-series} = \frac{m^3 - m}{12} = \frac{2^3 - 2}{12} = 0.5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5)$$

$$\text{Correction factors for } y\text{-series} = \frac{2^3 - 2}{12} + \frac{2^3 - 2}{12} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5)$$

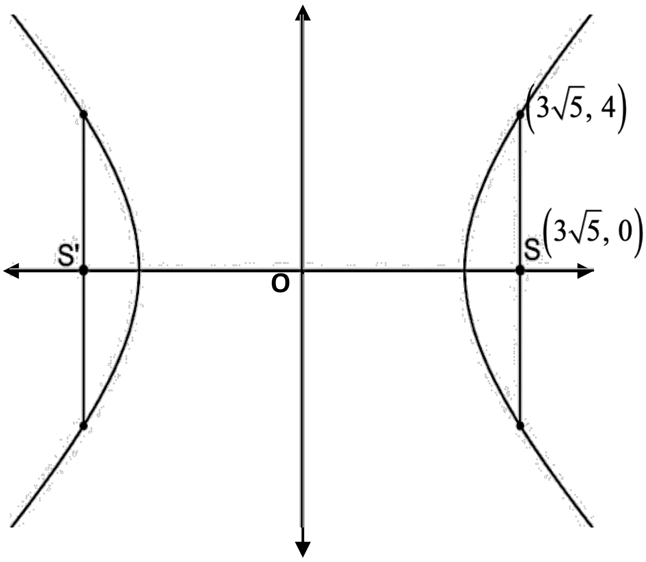
$$\text{Total correction factors} = 0.5 + 1 = 1.5.$$

$$\left.\begin{aligned} r &= 1 - \frac{6(\sum D^2 + 1.5)}{n(n^2 - 1)} \\ &= 1 - \frac{6(26 + 1.5)}{8(8^2 - 1)} \\ &= \frac{504 - 165}{504} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.5)$$

$$= 0.6726 \approx 0.67$$

Question 4

a) Find the equation of the conic section given below: **[3]**



Answer:

Given: $ae = 3\sqrt{5}, \quad \frac{2b^2}{a} = 8$. Since it is a horizontal hyperbola, the equation is

$$\left.\begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \end{array}\right\} \text{...(0.5)}$$

We have $b^2 = a^2(e^2 - 1) = a^2e^2 - a^2$..(0.5)

$$\left.\begin{array}{l} \Rightarrow 4a = (3\sqrt{5})^2 - a^2 \\ a^2 + 4a = 45 \\ a^2 + 4a - 45 = 0 \\ (a+9)(a-5) = 0 \\ \Rightarrow a = 5 \\ b^2 = 4(5) = 20 \end{array}\right\} \text{...(1)}$$

$\therefore$ The equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1 \ or \ 4x^2 - 5y^2 = 100$................(1)

b) Solve the system of equations using matrix method. **[4]**

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

Answer:

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$

$$\left.\begin{array}{l} |A| = (14-12)-(7-3)+(4-2)=0, \text{ either the system is inconsistent or} \\ \text{consistent with infinite solutions.} \end{array}\right\}..(0.5)$$

Co-factors of A

$$\left.\begin{array}{lll} a_{11}=2 & a_{21}=-3 & a_{31}=1 \\ a_{12}=-4 & a_{22}=6 & a_{32}=-2 \\ a_{13}=2 & a_{23}=-3 & a_{33}=1 \\ Adj.A = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix} & & \end{array}\right\}\text{...(1)}$$

$$\left.\begin{array}{l} Adj.A(B) = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ \therefore \text{ The system is consistent with infinite solutions.} \end{array}\right\}\text{..(1)}$$

Let $z = k$, where k is any real number

Substituting the value of z in equation (1) and (2), and solving simultaneosly:

$$\left.\begin{array}{l} x+y+k=6 \Rightarrow x+y=6-k \Rightarrow x=6-k-y \\ x+2y+3k=14 \Rightarrow x+2y=14-3k \\ \text{Substituting } x=6-k-y \text{ in } x+2y=14-3k, \\ 6-k-y+2y=14-3k \Rightarrow y=14-6-3k+k=8-2k \end{array}\right\}\text{.................................(1)}$$

$x = 6-k-8+2k = k-2$...(0.5)

Question 5

a) If $\sqrt{\dfrac{y}{x}} + \sqrt{\dfrac{x}{y}} = 4$, show that $\dfrac{dy}{dx} = \dfrac{7y-x}{y-7x}$. **[3]**

Answer:

$$\left.\begin{aligned}\sqrt{\frac{y}{x}}+\sqrt{\frac{x}{y}}=4 &\Rightarrow \frac{y+x}{\sqrt{xy}}=4\\ x+y&=4\sqrt{xy}\\ (x+y)^2&=16xy\\ x^2+2xy+y^2&=16xy\\ x^2+y^2&=14xy\end{aligned}\right\}\text{...(1)}$$

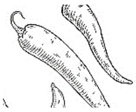
Differentiating on both sides implicitly,

$$\left.\begin{aligned}2x+2y\frac{dy}{dx}&=14x\frac{dy}{dx}+14y\\ 2y\frac{dy}{dx}-14x\frac{dy}{dx}&=14y-2x\\ (2y-14x)\frac{dy}{dx}&=14y-2x\end{aligned}\right\}\text{...(1.5)}$$

$$\left.\frac{dy}{dx}=\frac{2(7y-x)}{2(y-7x)}=\frac{7y-x}{y-7x}\right\}\text{...(0.5)}$$

b) *Ema-datshi* is prepared using the following ingredients:

Datshi



Chilies

Tomato

Oil

Coriander

Garlic

While preparing *Ema-datshi*, find the number of ways you can add the ingredients if:

i. coriander has to be always added at the end. **[1]**

Answer:

Since coriander is always added at the end, the remaining 5 indgredients can be added in 5!=120 ways.

ii. coriander is always added after *datshi*. **[1]**

Answer: Since coriander is always added after datshi, considering coriander and datshi as one ingredient, we have 5 ingredients, which can be added in $5!=120$ ways……(0.5)

Further, coriander and datshi can be added in 2 ways. Thus the total number of ways is $120\times 2=240$ ways……………………………………………………………………(0.5)	
iii. oil, datshi and coriander cannot be added at the same time.	**[2]**
Answer: Since the number of things not occurring together is three, the remaining three ingredients can be added in $3!=6$ ways………………………..…………………(0.5) Then the oil, cheese and coriander can be added in between the three things, first and last, in which there are four ways in which oil, cheese and coriander can be added. Thus, the number of ways they can be added is ${}^{4}P_{3}=24$………………………………(1) Therefore, the total number ways in which the oil, cheese and coriander cannot be added one after another is $6\times 24=144$ ways……………………………………………..(0.5)	
Question 6	
a) For a house-warming party, the host who is a male, invited 8 friends, comprising 3 males and 5 females. He arranged a circular table for them to be seated during the party.	
i. Find the probability that the host will always be seated with one particular female friend.	**[1.5]**
Answer: The total number of ways in which 9 persons can be sit around circular table: $(9-1)!=8!=40320$. ……………………………………………………………..(0.5) The number of ways in which two persons can always sit together: $(8-1)!\times 2!=7!2!=10080$………………………………………………………….(0.5) Therefore, the required probability is $\frac{10080}{40320}=\frac{1}{4}$ *or* 0.25.………………………..(0.5)	
ii. If two of them are randomly chosen to sing, what is the probability that the singers will be a male and a female?	**[1.5]**
Answer: Since there are 9 persons altogether, 2 persons can be chosen for singing in ${}^{9}C_{2}=36$ ways……………………………………………………………………………….(0.5) The number of ways that the singers will be a male and female is ${}^{4}C_{1}\times {}^{5}C_{1}=20$…(0.5) Therefore, the required probability is $\frac{20}{36}=\frac{5}{9}$ *or* 0.556……………………………..(0.5)	

b) Evaluate: $\int \frac{6x+15}{\sqrt{3x+8}}\,dx$. **[4]**

Answer:

$$\left.\begin{aligned}\int \frac{6x+15}{\sqrt{3x+8}}dx &= \int \frac{(6x+16)-1}{\sqrt{3x+8}}dx = \int\left(\frac{6x+16}{\sqrt{3x+8}} - \frac{1}{\sqrt{3x+8}}\right)dx \\ &= \int\left(\frac{2(3x+8)}{\sqrt{3x+8}} - \frac{1}{\sqrt{3x+8}}\right)dx \\ &= 2\int (3x+8)^{1/2}dx - \int (3x+8)^{-1/2}dx\end{aligned}\right\}\text{.................(2)}$$

$$= \frac{2(3x+8)^{3/2}}{3\left(\frac{3}{2}\right)} - \frac{(3x+8)^{1/2}}{3\left(\frac{1}{2}\right)} + C\text{...(1)}$$

$$= \frac{4}{9}(3x+8)^{3/2} - \frac{2}{3}(3x+8)^{1/2} + C\text{..(1)}$$

$$= \frac{2}{3}(3x+8)^{1/2}\left[\frac{2}{3}(3x+8)-1\right]+C$$

Question 7

a) Find n for each:

i. ${}^4P_2 = n\,{}^4C_2$ **[1.5]**

Answer:

$$\left.\begin{aligned}{}^4P_2 &= \frac{4!}{2!} \\ n\,{}^4C_2 &= \frac{4!n}{2!2!}\end{aligned}\right\}\text{...(0.5)}$$

$$\left.\begin{aligned}\frac{4!}{2!} = \frac{4!n}{2!2!} &\Rightarrow 4!2!n = 4!2!2! \\ &\Rightarrow n = 2\end{aligned}\right\}\text{...(1)}$$

ii. $P(n,\,6) = 3P(n,\,5)$ **[1.5]**

Answer:

$$\left.P(n,6)=\frac{n!}{(n-6)!},\quad 3P(n,5)=\frac{3n!}{(n-5)!}\right\}\text{...}(0.5)$$

$$\frac{n!}{(n-6)!}=\frac{3n!}{(n-5)!}\Rightarrow 3n!(n-6)!=n!(n-5)!$$

$$\left.\begin{aligned}&\Rightarrow\frac{3n!(n-6)!}{n!(n-5)!}=1\\&\Rightarrow\frac{3(n-6)!}{(n-5)(n-6)!}=1\\&\Rightarrow 3=n-5\\&\Rightarrow n=8\end{aligned}\right\}\text{..}(1)$$

b) Find $\frac{d^2y}{dx^2}$ of $y=x\sqrt{x-2}$. **[4]**

Answer:

$$\left.\begin{aligned}\frac{dy}{dx}&=x\frac{d}{dx}(\sqrt{x-2})+\sqrt{x-2}\frac{d}{dx}(x)\\&=\frac{x}{2\sqrt{x-2}}+\sqrt{x-2}\end{aligned}\right\}\text{....................................}(1)$$

$$\left.\begin{aligned}&=\frac{x+2x-4}{2\sqrt{x-2}}\\&=\frac{3x-4}{2\sqrt{x-2}}\end{aligned}\right\}\text{...}(0.5)$$

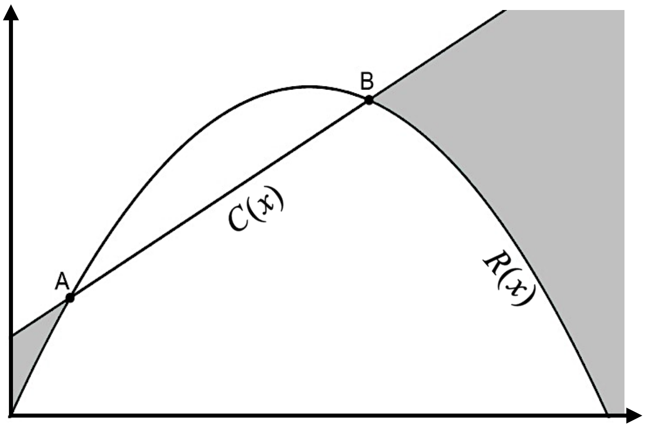
$$\left.\begin{aligned}\frac{d^2y}{dx^2}&=\frac{2\sqrt{x-2}\frac{d}{dx}(3x-4)-(3x-4)\frac{d}{dx}2\sqrt{x-2}}{\left(2\sqrt{x-2}\right)^2}\\&=\frac{6\sqrt{x-2}-\frac{3x-4}{\sqrt{x-2}}}{4(x-2)}\end{aligned}\right\}\text{.................}(1)$$

$$=\frac{6(x-2)-3x+4}{4(x-2)^{3/2}}\text{...}(1)$$

$$\left.\begin{aligned}&=\frac{6x-12-3x+4}{4(x-2)^{3/2}}\\&=\frac{3x-8}{4(x-2)^{3/2}}\end{aligned}\right\}\text{..}(0.5)$$

Question 8

a) The total cost function and the total revenue function of a certain company are represented in the graph below. **[3]**



Fill the table with appropriate terms associated with the graph.

a) Points A and B	**---------------------------------------**
b) The area between the two graphs from points A to B	**---**
c) The shaded areas	**-----------------------------------**

Answer:

a) Points A and B	**Break-even points**	**1 mark**
b) The area between the two graphs from points A to B	**Total Profit/Profit**	**1mark**
c) The shaded area	**Total Loss/Loss**	**1 mark**

b) Dophu claims that if we plot the points $(4, 7, 8), (2, 3, 4), (-1, -2, 1)$ and $(1, 2, 5)$ in 3-D coordinate system, the joining of the points will form a parallelogram. Verify the claim. **[4]**

Answer: A parallelogram is a shape in which opposite sides are parallel and equal.
Let the vertices of the parallelogram be $A(4, 7, 8), B(2, 3, 4), C(-1, -2, 1)$ and $D(1, 2, 5)$.
Checking the opposite sides, AB and CD

$$\left.\begin{array}{l}\text{Lengths of } AB = \sqrt{(4-2)^2+(7-3)^2+(8-4)^2} = 6 \text{ units} \\ \text{Lengths of } CD = \sqrt{(-1-1)^2+(-2-2)^2+(1-5)^2} = 6 \text{ units} \\ \textit{Therefore, the two opposite sides are equal.}\end{array}\right\} \text{..................................(1)}$$

$$\left.\begin{array}{l}\text{Direction ratios of } AB: a_1 = 4-2 = 2,\ b_1 = 7-3 = 4,\ c_1 = 8-3 = 4 \\ \text{Direction ratios of } CD: a_2 = -1-1 = -2,\ b_2 = -2-2 = -4,\ c_2 = 1-5 = -4 \\ \frac{a_1}{a_2} = \frac{2}{-2} = -1,\ \frac{b_1}{b_2} = \frac{4}{-4} = -1,\ \frac{c_1}{c_2} = \frac{4}{-4} = -1 \\ \text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}, \text{ the sides AB and CD are parallel.}\end{array}\right\}\text{.......(1)}$$

Checking the opposite sides, BC and AD

$$\left.\begin{array}{l}\text{Lengths of } BC = \sqrt{(2+1)^2 + (3+2)^2 + (4-1)^2} = \sqrt{43} \text{ units} \\ \text{Lengths of } AD = \sqrt{(4-1)^2 + (7-2)^2 + (8-5)^2} = \sqrt{43} \text{ units} \\ \textit{Therefore, the two opposite sides are equal.}\end{array}\right\}\text{....................................(1)}$$

$$\left.\begin{array}{l}\text{Direction ratios of } BC: a_1 = 2+1 = 3,\ b_1 = 3+2 = 5,\ c_1 = 4-1 = 3 \\ \text{Direction ratios of } AD: a_2 = 4-1 = 3,\ b_2 = 7-2 = 5,\ c_2 = 8-5 = 3 \\ \frac{a_1}{a_2} = \frac{3}{3} = 1,\ \frac{b_1}{b_2} = \frac{5}{5} = 1,\ \frac{c_1}{c_2} = \frac{3}{3} = 1 \\ \text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}, \text{ the sides BC and AD are parallel.}\end{array}\right\}\text{.......(1)}$$

Therefore, the given points are the vertices of a parallelogram.

Question 9

a) The equation of conic section is $y^2 + 6x = 0$. Graph the conic section, and show its focus, directrix and the equation of directrix. **[3]**

Answer: The equation $y^2 + 6x = 0$ is an equation of parabola.

$y^2 + 6x = 0$

$y^2 = -6x$

$y^2 = -4\left(\frac{3}{2}\right)x,\ \left[\because y^2 = -4ax\right]$

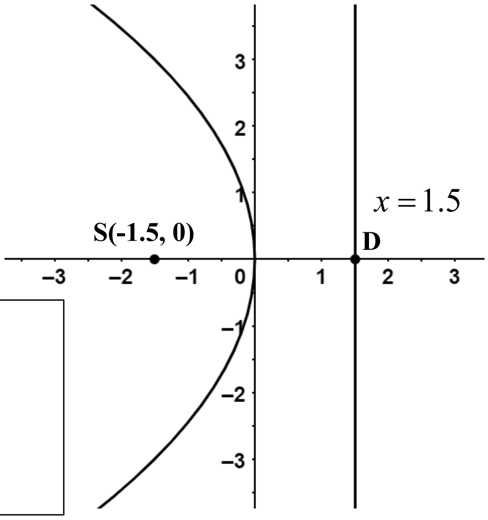
$a = \frac{3}{2}$ or 1.5

Drawing graph accurately: 1 mark

Representing focus: 1 mark

Directrix line: 0.5 mark

Equation of directrix: 0.5



b) The selection process for certain scholarships involves ranking students according to their academic marks and interview scores. The table provided below is the ranking of six students who got selected in the scholarships. Determine the line of regression of *x* on *y*. **[4]**

Academic (70%): *x*	57	56	56	54	54	53
Interview (30%): *y*	21	20	22	18	20	19

Answer:

x	***y***	***x*²**	***y*²**	***Xy***
57	21	3249	441	1197
56	20	3136	400	1120
56	22	3136	484	1232
54	18	2916	324	972
54	20	2916	400	1080
53	19	2809	361	1007
330	**120**	**18162**	**2410**	**6608**

$\sum x = 330,\ \sum y = 120,\ \sum x^2 = 18162, \sum y^2 = 2410,\ \sum xy = 6608$……….(1.5)

$$\bar{x} = \frac{330}{6} = 55,\quad \bar{y} = \frac{120}{6} = 20 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.5)$$

$$\left.\begin{aligned} b_{xy} &= \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}} \\ b_{xy} &= \frac{6608 - \frac{(330)(120)}{6}}{2410 - \frac{120^2}{6}} = \frac{8}{10} = \frac{4}{5} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Regression equation of x on y is $(x - \bar{x}) = b_{xy}(y - \bar{y}$

$$\left.\begin{aligned} (x - 55) &= \frac{2}{5}(y - 20) \\ 5x - 275 &= 2y - 40 \\ 5x - 2y - 275 + 40 &= 0 \\ 5x - 2y - 235 &= 0 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Question 10

a) The sum of two positive real numbers is 20.5, and the sum of their squares is minimum. Determine the numbers. **[3]**

Answer:

$$\left.\begin{array}{l}\text{Let the numbers be } x \text{ and } y\text{. Then}\\ x+y=20.5\\ y=20.5-x\end{array}\right\}\text{...(0.5)}$$

Sum of their squares: $S=x^2+(20.5-x)^2$..(0.5)

$$\left.\begin{array}{l}S=x^2+420.25-41x+x^2\\ S=2x^2-41x+420.25\end{array}\right\}\text{..(1)}$$

$$\left.\begin{array}{l}S'=4x-41\\ S'=0\Rightarrow 4x-41=0\Rightarrow x=10.25\\ S''=4>0,\therefore \text{ There is a minimum point at } x=10.25.\\ \therefore \text{The numbers are 10.25 and 10.25}\end{array}\right\}\text{...................................(1)}$$

b) A young entrepreneur who wishes to start a new business venture obtained a loan of Nu 400,950 from a bank under the condition of repaying it with compound interest at 6% p.a. The repayment plan requires making annual instalments of Nu 150,000 each. Determine the number of payments the entrepreneur must make in order to settle the debt. **[4]**

$P=400950,\ \ a=150000,\ i=\dfrac{6}{100}=0.06,\ n=?$...(0.5)

$P=\dfrac{a}{i}\left[1-(1+i)^{-n}\right]$...(0.5)

$$\left.\begin{array}{l}400950=\dfrac{150000}{0.06}\left[1-1.06^{-n}\right]\\ 400950=2500000(1-1.06^{-n})\\ 1-1.06^{-n}=0.10638\\ -1.06^{-n}=-0.83962\\ 1.06^{-n}=0.83962\end{array}\right\}\text{...(1.5)}$$

$$\left.\begin{array}{l}-n\log 1.06=\log 0.83962\\ -n=\dfrac{\log 0.83962}{\log 1.06}=-2.99\\ n\approx 3.\end{array}\right\}\text{...(1)}$$

Answer: Given:

$\therefore$ The entrepreneur needs to make 3 payments to settle his debt..................(0.5)

Question 11

a) A businessman accepted a bill of Nu 1,500,000 on November 4, 2023, which was payable after 3 months. The drawer discounted the bill on November 20, 2023 from a bill broker at 6.5% p.a. Compute the amount the drawer got from the bill broker. **[3]**

Answer:

Bill accepted on: November 4, 2023 for 3 months.

Maturity date (Due date): February 4, 2024

Discounted on: November 20, 2023,

$$\left.\begin{array}{l}\text{Given: } A=1500000,\ i=\dfrac{6.5}{100}=0.065\\ n=\dfrac{79}{365}\end{array}\right\}\text{.................................(1)}$$

Banker's Discount: Ani...(0.5)

$$\left.\begin{array}{l}=1500000\left(\dfrac{79}{365}\right)(0.065)\\ =21102.74\end{array}\right\}\text{......................(1)}$$

Month	Days
$Nov(30)$	10
$Dec(31)$	31
$Jan(31)$	31
Feb	4
$Grace$	3
$Total$	$79 days.$

$\therefore$ The value of the bill (the amount the broker gave to the drawer):

$1500000-21102.74=$ Nu 1,478,897.26.........(0.5)

b) Evaluate: $\int \frac{x-1}{(x+1)(x^2+1)}dx$ **[4]**

Answer:

$$\left.\begin{array}{l}\dfrac{x-1}{(x+1)(x^2+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}\\ \Rightarrow A(x^2+1)+(Bx+C)(x+1)=x-1\end{array}\right\}\text{...(0.5)}$$

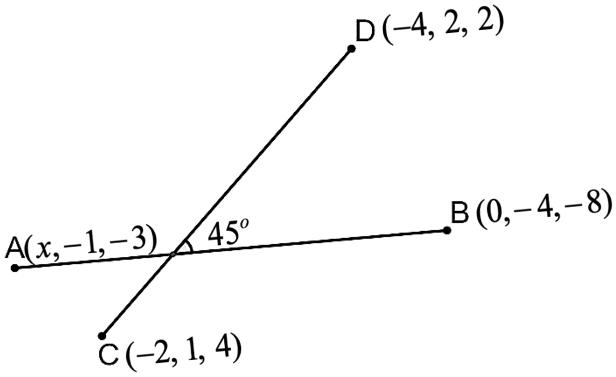
$$\left.\begin{array}{l}\text{Put } x=-1,\ 2A=-2\Rightarrow A=-1.\\ \text{Coefficients of } x^2: A+B=0\Rightarrow -1+B=0\Rightarrow B=1\\ \text{Constants: } A+C=-1\Rightarrow -1+C=-1\Rightarrow C=0.\\ \dfrac{x-1}{(x+1)(x^2+1)}=\dfrac{-1}{x+1}+\dfrac{x}{x^2+1}\end{array}\right\}\text{..................................(1)}$$

$$\left.\begin{array}{l}\therefore \displaystyle\int\frac{x-1}{(x+1)(x^2+1)}dx=\int\left(\frac{-1}{x+1}+\frac{x}{x^2+1}\right)dx\\ \qquad\qquad\qquad\qquad\displaystyle=-\int\frac{1}{x+1}dx+\int\frac{x}{x^2+1}dx\\ \text{Using substitution for } \displaystyle\int\frac{x}{x^2+1}dx:\\ \text{Let } u=x^2+1,\ \dfrac{du}{dx}=2x\Rightarrow\dfrac{1}{2}du=xdx\end{array}\right\}\text{...(1)}$$

$$\int\frac{x}{x^2+1}dx=\frac{1}{2}\int\frac{1}{u}du=\frac{1}{2}\log|u|+C=\frac{1}{2}\log\left|x^2+1\right|+C\text{...............................(0.5)}$$

$$\therefore\ -\int\frac{1}{x+1}dx+\int\frac{x}{x^2+1}dx=-\log|x+1|+\frac{1}{2}\log\left|x^2+1\right|+C\text{..........................(1)}$$

Question 12

a) Find the value of x in the diagram. 	**[3]**

Answer: Since the coordinates and the angle between the lines are given, we have to use angle between two lines using the direction ratios:

Direction ratios of line 1:

$a_1 = x - 0 = x,\ b_1 = -1 - (-4) = 3,\ c_1 = -3 - (-8) = 5$................................(0.5)

Direction ratios of line 2:

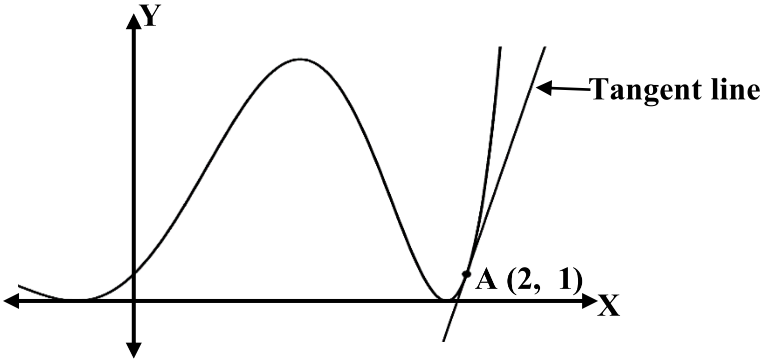
$a_2 = -2 - (-4) = 2,\ b_2 = 1 - 2 = -1,\ c_2 = 4 - 2 = 2$.....................................(0.5)

$$\text{Cos}\, 45^o = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$\left.\begin{array}{l} \frac{1}{\sqrt{2}} = \frac{2x + 3(-1) + 5(2)}{\sqrt{x^2 + 3^2 + 5^2}\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{2x+7}{\sqrt{x^2+34}\sqrt{9}} = \frac{2x+7}{3\sqrt{x^2+34}} \\ \frac{1}{2} = \frac{(2x+7)^2}{9(x^2+34)} = \frac{4x^2 + 28x + 49}{9x^2 + 306} \\ 9x^2 + 306 = 8x^2 + 56x + 98 \\ x^2 - 56x + 208 \end{array}\right\}\text{..........(1)}$$

$$\left.\begin{array}{l} x = \frac{-(-56) \pm \sqrt{(-56)^2 - 4(208)}}{2} \\ x = \frac{56 \pm 48}{2} \\ x = 52 \text{ and } 4. \end{array}\right\}\text{..(1)}$$

b) In a certain town, it is observed that 85% of the population speaks *Dzongkha*, 40% speaks *Tshangla*, and 20% speaks *Khengkha*. Furthermore, 32% of the population can speak both *Tshangla* and *Dzongkha*, while 13% can speak *Dzongkha* and *Khengkha*, and 10% can speak *Tshangla* and *Khengkha*.	
i. If a person is selected randomly from the town, what is the probability of the person speaking all the three languages?	**[2.5]**
Answer: Let P(A), P(B) and P(C) be the probabilities of the people speaking Dzongkha, Tshangla and Khengkha respectively. Then $\left.\begin{array}{l} P(A)=85\%=0.85, \quad P(B)=40\%=0.4, \quad P(C)=20\%=0.2 \\ P(A\cap B)=32\%=0.32,\ P(A\cap C)=13\%=0.13,\ P(B\cap C)=10\%=0.1 \end{array}\right\}....(0.5)$ Probability of the person speaking all the languages is $P(A\cap B\cap C)$. We have, $\left.\begin{array}{l} P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C) \\ \qquad\qquad +P(A\cap B\cap C) \end{array}\right\}.....(0.5)$ $1=0.85+0.4+0.2-0.32-0.13-0.1+P(A\cap B\cap C)$.......................................(0.5) $P(A\cap B\cap C)=1-0.9=0.1$..(1)	
ii. If a person is randomly selected who speaks Khengkha, what is the probability that the person also speaks Tshangla?	**[1.5]**
$P(B)=0.4,\ P(C)=0.2$..............................(0.5) Answer: Given $P(B/C)=\dfrac{P(B\cap C)}{P(C)}$.................................(0.5) $P(B/C)=\dfrac{0.1}{0.2}=0.5$....................................(0.5)	
Question 13	
a) Find the slope of the tangent line to the curve at point A if the equation of the curve is $y=(3x-x^3+1)^2$.	**[3]**

	
Answer: Given the equation of the curve: $y=(3x-x^3+1)^2$ $\frac{dy}{dx}=2(3x-x^3+1)(3-3x^2)$...........................(1.5) $\left.\frac{dy}{dx}\right\vert_{x=2}=2(2\times 2-2^3+1)(3-3\times 2^2)=18$........(1.5)	
b) In BHSEC Mathematics Examination, students must answer 11 out of 13 questions. Find the number of choices a student has if:	
i. the first question is compulsory.	**[1]**
Answer: Since the first question is compulsory, the remaining 10 questions can be chosen from 12 questions in ${}^{12}C_{10}=66$ ways	
ii. they have to answer at least five questions from the first six questions.	**[1.5]**
Answer: The number of possible ways: 5 from the first 6, and 6 from the remaining 7 : ${}^{6}C_5\times{}^{7}C_6=42$.................(0.5) 6 from the first 6 and 5 from the remaining 7: ${}^{6}C_6\times{}^{6}C_5=6$......................(0.5) $\therefore$ Total possible ways: $42+6=48$..(0.5)	
iii. they have to answer exactly two questions from the last three questions.	**[1.5]**
Answer: The number of possible choices are: selecting two from the last three questions and 9 from the remaining 10 questions. ${}^{3}C_2\times{}^{10}C_9=3\times 10=30$ ways……………………………………………[1.5]	