BHUTAN HIGHER SECINDARY EDUCATION CERTIFICATE
DECEMBER-2023
MODEL ANSWER & MARKING SCHEME – BUSINESS MATHEMATICS

GENERAL INSTRUCTIONS

1) The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the marking scheme are suggested answers. The content is thus indicative. If a student has given any other answer, which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weighting.

2) Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and strictly followed once the answers are thoroughly standardized.

3) If a question has parts, please award marks in the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin at the end of question. If a question does not have any parts, marks must be awarded in the left-hand margin.

4) If a candidate has attempted an extra question, it will be cancelled. The extra question attempted will be the last question in the sequence of the question paper.

5) Some examinees may attempt the questions giving equally correct answers in a different way. If the examiners are convinced that the response given by an examinee is genuinely correct, full weighting should be given.

6) If there are questions on distinction between two concepts, in such questions, sometimes some students give one aspect of the difference correctly and the other is either wrong or not given at all, no marks should be awarded.

7) There may be some questions requiring the examinees to give new ideas of their own or pass their own judgments and give valid justifications. In such cases, marks should be awarded for their efforts though there may be several possible answers.

8) If the questions ask for two features/characteristics/points but the examinee writes more than two features/characteristics/points, say, five of which the first is correct, second is incorrect, the best two should be assessed and the remaining should be ignored.

9) It is expected that the Marking Scheme be followed objectively for reliable marking. For instance, if an examinee scores 35 (BCSE) / 35 (BHSEC) marks, his/her marks should not be inflated to 40 (BCSE) / 40 (BHSEC) simply to pass him/her. Similarly, whenever an examinee has answered the question effectively, his/her marks should not be deducted unnecessarily. Do not hesitate to award full marks if the answer deserves it.

10) Marks should be awarded keeping in view the total marks of that particular question and not the total marks of the question paper. For example, if 1 mark is given to a question carrying 3 marks, even if nothing is correct, then that 1 mark constitutes 33% of the total marks ear-marked for this answer. This must be avoided. If a candidate fails in the subject by 1 to 3 marks, the marking may be reviewed.

SECTION A [30 MARKS]
ANSWER ALL QUESTIONS

Question 1: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choices circled, NO score will be awarded. **[30]**

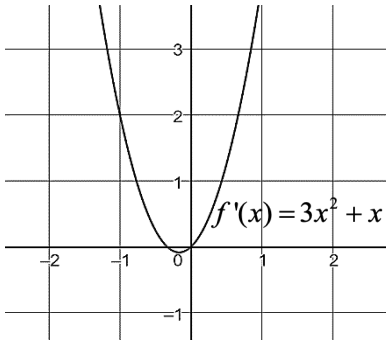
i) The insurance scheme where a certain amount is paid until a child completes class XII is

(A) Annuity contingent.
B Annuity perpetual.
C Annuity deferred.
D Annuity certain.

Solution:
Annuity contingent since the number of years to complete class XII is not fixed and uncertain.

ii) The derivative of a function is shown in the figure. Find its function.

A $f(x) = 6x + 1 + c$
B $f(x) = x^3 + x^2 + c$
C $f(x) = 3x^3 + x^2 + c$
(D) $\boldsymbol{f(x) = x^3 + \frac{x^2}{2} + c}$



Solution:

Given $f'(x) = 3x^2 + x$

$f(x) = \int (3x^2 + x)\,dx = x^3 + \frac{x^2}{2} + c$

Since the point (-1, 2) lies on the graph,

$f(-1) = (-1)^3 + \frac{(-1)^2}{2} + c = 2$

$\Rightarrow -1 + \frac{1}{2} + c = 2$

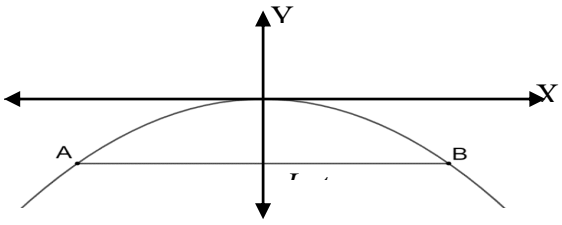
$\Rightarrow c = 2 + \frac{1}{2}$

$\Rightarrow c = \frac{5}{2}$

$\therefore f(x) = x^3 + \frac{x^2}{2} + \frac{5}{2}$

iii) The equation of the conic section given in the figure is $-x^2 = 2y$. Calculate the length AB.

(A) 2 units
B 4 units
C 6 units
D 8 units



Solution:

Given the equation: $-x^2 = 2y$

$\because -x^2 = 4ay,\ 4a = 2,\ \therefore a = \frac{2}{4} = \frac{1}{2}$

The length AB(Latus rectum): $4a = 4 \times \frac{1}{2} = 2$ units.

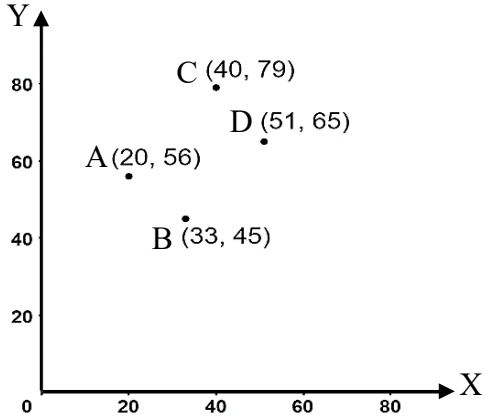
iv) The graph depicts the comparison of the Internal Energy Sale in million Ngultrums of Kurichhu Hydro Project and Tala Hydro Project for the year 2012 to 2015. Compute the correlation between the two hydro projects.

A –0.46

B –0.04

(C) 0.46

D 0.48



Solution:

x	y	xy	x^2	y^2
20	56	1120	400	3136
33	45	1485	1089	2025
40	79	3160	1600	6241
51	65	3315	2601	4225
144	**245**	**9080**	**5690**	**15627**

$\sum x = 144,\ \sum y = 245,\ \sum xy = 9080,\ \sum x^2 = 5690,\ \sum y^2 = 15627$

$$\therefore r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}}$$

$$r = \frac{4(9080) - (144)(254)}{\sqrt{\left(4(5690) - 144^2\right)\left(4(15627) - 245^2\right)}} = 0.46394 \approx 0.46$$

v) If the co-factor (a_{32}) of the matrix $\begin{bmatrix} x & 0 & 2 \\ 3 & -5 & 4 \\ 1 & 5 & 1 \end{bmatrix}$ is 10, evaluate the value of x.

A –4

(B) **–1**

C 1

D 4

Solution:

Given: $\begin{bmatrix} x & 0 & 2 \\ 3 & -5 & 4 \\ 1 & 5 & 1 \end{bmatrix}$

$$a_{32} = (-1)^{3+2}\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = 10$$

$$\Rightarrow -(4x-6) = 10$$

$$\Rightarrow -4x + 6 = 10$$

$$\Rightarrow -4x = 4$$

$$\Rightarrow x = -1$$

vi) The total cost and the total revenue graphs are given in the figure. From the figure, find the points A and B.

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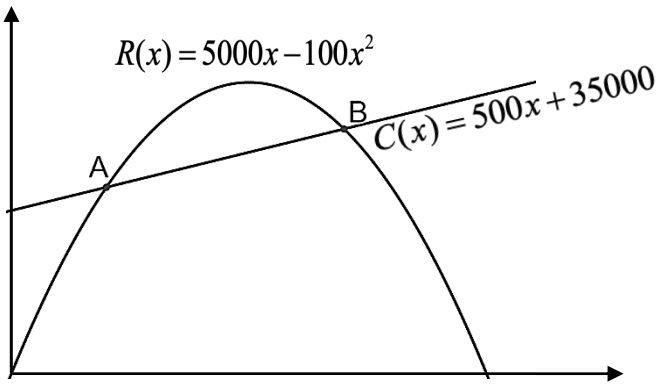
A $x = -10, x = -35$

B $x = -10, x = 35$

C **$x = 10,\ x = 35$**

D $x = 10, x = -35$



Solution:

Break-even Points: $C(x) = R(x) \Rightarrow 500x + 35000 = 5000x - 100x^2$

$$\Rightarrow 100x^2 - 5000x + 500x + 35000 = 0$$

$$\Rightarrow 100x^2 - 4500x + 35000 = 0$$

$$\Rightarrow x^2 - 45x + 350 = 0$$

$a = 1,\ b = -45,\ c = 350$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{45 \pm \sqrt{45^2 - 4(1)(350)}}{2(1)}$$

$$x = \frac{45 \pm 25}{2}$$

$x = 10, \quad x = 35.$

vii) If a line has the direction angles $\beta = 60^o$ and $\gamma = 30^o$, find the angle α.

A 30^o

B 45^o

C 60^o

(D) **90^o**

Solution:

From the relation: $l^2 + m^2 + n^2 = 1$,

$(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2 = 1$

$\cos^2\alpha + \cos^2 60^o + \cos^2 30^o = 1$

$\cos^2\alpha + \frac{1}{4} + \frac{3}{4} = 1$

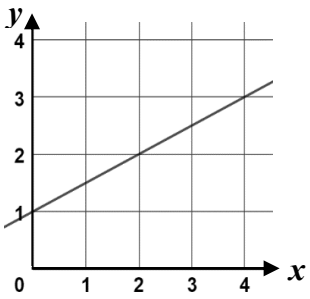
$\cos^2\alpha = 1 - 1$

$\alpha = \cos^{-1}(0) = 90^o$

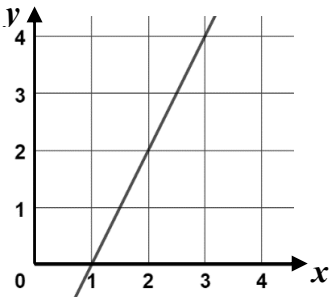
viii) The table given shows the sales of two different types of commodities.

x	2	3	4	5
y	2	4	6	8

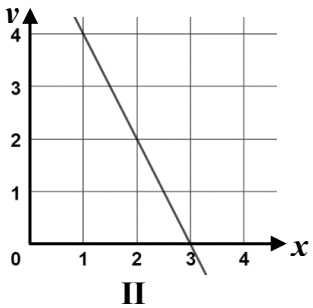
Which of the following graphs represents the regression line of '***x*** on ***y***' for the above data?



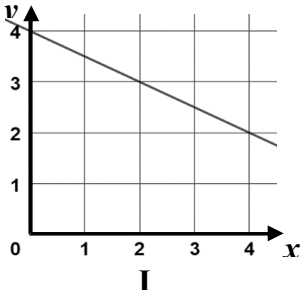
I



II



II



I

A I

(B) II

C III

D IV

Solution:

x	y	xy	y^2
2	**2**	**4**	**4**
3	**4**	**12**	**16**
4	**6**	**24**	**36**
5	**8**	**40**	**64**
$\sum x = 14$	$\sum y = 20$	$\sum xy = 80$	$\sum y^2 = 120$

The regression coefficient, $b_{xy} = \dfrac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$

$$b_{xy} = \frac{4(80) - 14(20)}{4(120) - 20^2} = \frac{1}{2}$$

Since the graph II has the slope of $\frac{1}{2}$, it represents the regression line of x on y.

ix) A calculator is marked at Nu 1250 with successive discounts of 20% and 10%. An additional discount of 5% is allowed during the payment. Find the selling price of the calculator.

(A) Nu 855
B Nu 831
C Nu 812
D Nu 790

Solution:

Given $MP = Nu\,1250$
Less 1st discount: $0.8 \times 1250 = 1000$
Less 2nd discount: $0.9 \times 1000 = 900$
Less 3rd discount: $0.95 \times 900 = 855$

Therefore, the final selling price is **Nu 855**

x) Evaluate the differential coefficient of $y^2 = 4ax + y$ at $a = 2$ and $y = 1$.

(A) 8
B $\frac{1}{8}$
C $-\frac{1}{6}$
D -6

Solution:

Given: $y^2 = 4ax + y$

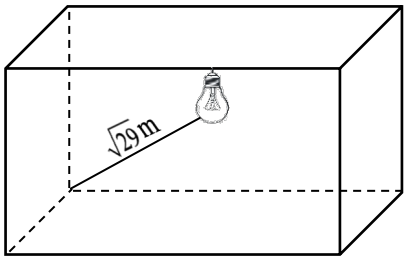
$$2y\frac{dy}{dx} = 4a + \frac{dy}{dx}$$

$$(2y-1)\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y-1}$$

$$\therefore \frac{dy}{dx} = \frac{4(2)}{2(1)-1} = 8.$$

xi) The light bulb shown in the diagram is anchored 2 m away from the left wall and 3m away from the front wall. How high is the bulb from the floor if the distance from the corner to the light bulb is $\sqrt{29}$ m.



A 16.0 m

B 6.5 m

C 4.3 m

(D) 4.0 m

Solution:

The coordinates of the bulb: (2, 3, z)

Distance: $\sqrt{2^2 + 3^2 + z^2} = \sqrt{29}$

$\Rightarrow 13 + z^2 = 29 \Rightarrow z^2 = 16$

$\therefore z = \sqrt{16} = 4$ metres.

xii) A student takes an International English Language Testing System (IELTS) examination. He estimates his chance of passing in listening at $\frac{4}{5}$, writing at $\frac{3}{4}$ and speaking at $\frac{2}{3}$. To qualify, he must pass in at least two areas. What is the probability that he qualifies?

A $\frac{13}{60}$

(B) $\mathbf{\frac{13}{30}}$

C $\frac{2}{5}$

D $\frac{4}{5}$

Solution:

Let $P(L), P(W)$ and $P(S)$ be the probability of passing in listening, writing and speaking respectively.

Given: $P(L)=\frac{4}{5}, P(W)=\frac{3}{4},\ P(S)=\frac{2}{3}.$

Probability that he qualifies: $P(L)P(W)P(\overline{S})+P(L)P(\overline{W})P(S)+P(\overline{L})P(W)P(S)$

$$=\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}$$

$$=\frac{1}{5}+\frac{2}{15}\times\frac{1}{10}$$

$$=\frac{13}{30}$$

xiii) Suppose you purchase a plot of land which is worth Nu 1 million, paying 50% in cash, and agreeing to pay the remaining amount on instalment basis half-yearly for five years. If the land owner charges an interest rate of 10% p.a., find the amount of each instalment.

A Nu 39,751.90
B Nu 61,668.85
(C) Nu 64,752.29
D Nu 81,372.69

Solution:

Given $P=500,000,\ i=\frac{0.1}{2}=0.05,\ n=5\times2=10$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]\Rightarrow a=\frac{Pi}{1-(1+i)^{-n}}$$

$$\therefore a=\frac{500000\times0.05}{1-1.05^{-10}}=64,752.29$$

xiv) In a bookshelf, 3 books of English, 3 books of Dzongkha and 4 books of science are to be arranged in such a way that the books of same subject are to be together. Find in how many ways this can be done?

A 288
B 864
C 1728
(D) 5184

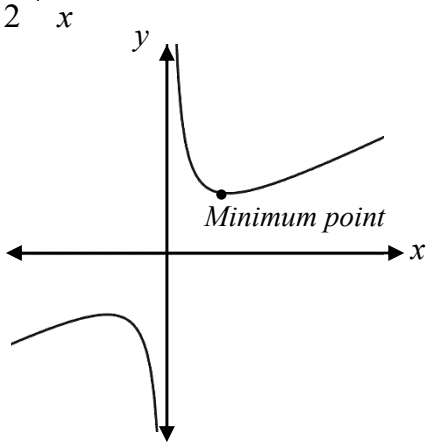
Solution:

Given the three different books, they can be arranged in 3! ways.

Since same subjects are to be together, they can be arranged among themselves in $3!, 3!$ and $4!$respectively.

Therefore, the total number of permutations: $3! \times 3! \times 3! \times 4! = 5184$.

xv) The equation of the given curve is $y = \frac{x}{2} + \frac{2}{x}$. Find the coordinates of the point indicated.



A (3, 2)

B (3, 3)

C (2, 2)

D (2, 3)

Solution:

Given the equation: $y = \frac{x}{2} + \frac{2}{x}$,

$\frac{dy}{dx} = \frac{1}{2} - \frac{2}{x^2}$

$\frac{dy}{dx} = 0 \Rightarrow \frac{1}{2} - \frac{2}{x^2} = 0$

$x^2 - 4 = 0$

$x^2 = 4 \Rightarrow x = \sqrt{4} = \pm 2.$

$\frac{d^2y}{dx^2} = \frac{4}{x^3}$

When $x = 2, \frac{d^2y}{dx^2} > 0, \therefore$ There is a minumum point at $x = 2$.

$y = \frac{2}{2} + \frac{2}{2} = 2$

$\therefore$ The coordinates of the point is (2, 2)

SECTION B [70 MARKS]
ATTEMPT ANY TEN QUESTIONS

Question 2

a) Pema bought 3 notebooks and 2 scrapbooks, and paid Nu 260. You purchased 2 notebooks and 3 scrapbooks, and paid Nu 240. Find the price of each item using Martin's rule. **[3]**

Solution:

Given

Let x and y be the price of one notebook and scrapbook respectively. Then

$3x + 2y = 260$

$2x + 3y = 240$

$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 260 \\ 240 \end{bmatrix}$

$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 260 \\ 240 \end{bmatrix}$

$X = A^{-1}B$

$|A| = 9 - 4 = 5 \neq 0 \therefore A^{-1}$ exists.....[0.5]

$Adj.A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} Adj.A = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$ .....[1]

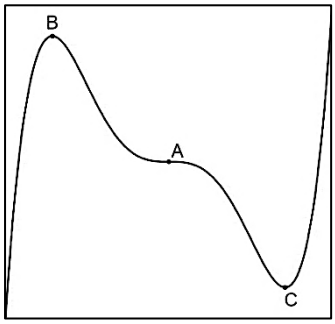
$X = A^{-1}B \Rightarrow \frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 260 \\ 240 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 300 \\ 200 \end{bmatrix}$

$= \begin{bmatrix} 60 \\ 40 \end{bmatrix}$ .....[1]

$\therefore$ The price of each book is Nu 60 and Nu 40 respectively.....[0.5]

b) Identify and find the coordinates of the points, A, B and C of the given curve $f(x)=\frac{1}{5}x^5-3x^3+1$. **[4]**



Solution:

Given: $f(x)=\frac{1}{5}x^5-3x^3+1$

$f'(x)=x^4-9x^2$

$f'(x)=0 \Rightarrow x^4-9x^2=0$

$\Rightarrow x^2(x^2-9)=0$

$\Rightarrow x=0,\ 3,\ -3$ .....(1)

$f''(x)=4x^3-18x$

$f''(0)=0,$

$f''(3)=4(3)^3-18(3)=54>0,$ so, there is a minimum point at $x=3.$

$f(3)=\frac{1}{5}(3)^5-3(3)^3+1=\frac{243}{5}-81+1=\frac{243-405+5}{5}=\frac{-157}{5}$ .....(1)

$\therefore$ The point C is minimum: $\left(3,\ \frac{-157}{5}\right)$

$f''(-3)=4(-3)^3-18(-3)=-54<0,$ so, there is a maximum point $x=-3.$

$f(-3)=\frac{1}{5}(-3)^5-3(-3)^3+1=\frac{-243}{5}+81+1=\frac{-243+405+5}{5}=\frac{167}{5}$ .....(1)

$\therefore$ The point B is maximum: $\left(-3,\ \frac{167}{5}\right)$

$f'''(x)=12x^2-18$

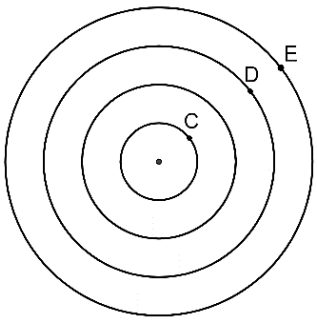
$f'''(0)=-18\neq 0,$ so, there is an inflexion point at $x=0.$

$f(0)=\frac{1}{5}0^5-3(0)^3+1=1.$ .....(1)

$\therefore$ The point A is inflexion: $(0,\ 1)$

Question 3

a) The points C(9, 8, -10), D(x, y, z) and E(3, 2, -4) are in a straight line as shown in the diagram. Find the coordinates of the point D. **[3]**



Solution:

Given: C(9, ,8, -10), D(x, y, z), E(3, 2, -10)

From the diagram, the point D divides CE in the ratio 2:1

$m_1 = 2,\ m_2 = 1$ .....[1]

The coordinates of point $D(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$

$= \left(\frac{2(3)+1(9)}{2+1}, \frac{2(2)+1(8)}{2+1}, \frac{2(-4)+1(-10)}{2+1}\right)$

$= \left(\frac{15}{3}, \frac{13}{3}, \frac{-18}{3}\right)$

$= (5, 4, -6)$ .....[2]

b) The table shows the marks of Continuous Assessment (CA) and Written Examination (WE) obtained by eight students. **[4]**

CA	19	18	19	17	20	18	20	17
WE	70	75	68	80	70	78	69	85

Evaluation coefficient and interpret the result.

Solution:

CA (x)	**WE (y)**	***xy***	***x²***	***y²***
19	70	1330	361	4900
18	75	1350	324	5625
19	68	1292	361	4624
17	80	1360	289	6400
20	70	1400	400	4900
18	78	1404	324	6084
20	69	1380	400	4761
17	85	1445	289	7225
148	**595**	**10961**	**2748**	**44519**

Table……..[1.5]

Here, $n=8,\ \sum x=148, \sum y=595,\ \sum xy=10961,\ \sum x^2=2748,\ \sum y^2=44519$

$$r=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-\left(\sum x\right)^2}\sqrt{n\sum y^2-\left(\sum y\right)^2}}$$

$$r=\frac{8(10961)-(148)(595)}{\sqrt{8(2748)-\left(148\right)^2}\sqrt{8(44519)-\left(595\right)^2}}$$

$$r=\frac{87688-88060}{\sqrt{80}\sqrt{2127}}$$

$$r=\frac{-372}{\sqrt{170{,}160}}$$

$$r=\frac{-372}{412.5045}$$

$$r=-0.9018$$

.....[2]

$\therefore$ The correlation is high degree negative.....[0.5]

Question 4

a) Differentiate $(2x+1)\sqrt{x}$ with respect to x. **[3]**

Solution:

$$\frac{d}{dx}\left((2x+1)\sqrt{x}\right)=\frac{d}{dx}\left(2x^{3/2}+x^{1/2}\right)$$

$$=2\times\frac{3}{2}x^{1/2}+\frac{1}{2}x^{-1/2}$$ .....[1]

$$=3\sqrt{x}+\frac{1}{2\sqrt{x}}$$ .....[1]

$$=\frac{6x+1}{2\sqrt{x}}$$ .....[1]

OR

$$\frac{d}{dx}\left((2x+1)\sqrt{x}\right)=(2x+1)\frac{d}{dx}(\sqrt{x})+\sqrt{x}\frac{d}{dx}(2x+1)$$.....[1]

$$\left.\begin{aligned}&=\frac{(2x+1)}{2\sqrt{x}}+2\sqrt{x}\\&=\frac{2x+1+4x}{2\sqrt{x}}\end{aligned}\right\}$$.....[1]

$$=\frac{6x+1}{2\sqrt{x}}$$.....[1]

b) The total cost per semester to study in Norbuling Rigter College is shown below. You are planning to study in the college after class XII as a boarder by saving Nu 8,000 per month in a bank that pays an interest rate of 7.5% p.a., How many years would it take to accumulate the amount that would cover the cost for six semesters? **[4]**

Fees per Semester		
Type	**Facilities**	**Amount**
Boarder	1. Tuition (also includes computer labs, library and other academic facilities relevant to the students with free on-campus internet connectivity) 2. Food 3. Room rentals	**Nu 84,734**
Days Scholar	1. Tuition (also includes computer labs, library and other academic facilities relevant to the students with free on-campus internet connectivity) 2. Food (Compulsory Lunch for day students)	**Nu 62,066**

Solution:

The total cost for six semesters = $6 \times 84734 = \text{Nu } 508,404$.....[0.5]

$A = 508,404,\ i = \dfrac{0.075}{12} = 0.00625,\ a = 8000,\ n = ?$

$A = \dfrac{a}{i}(1+i)\left((1+i)^n - 1\right)$

$508,404 = \dfrac{8000}{0.00625}(1.00625)\left(1.00625^n - 1\right)$.....[1]

$$1.00625^n - 1 = \frac{508,404 \times 0.00625}{8000 \times 1.00625}$$
$$1.00625^n - 1 = \frac{3177.525}{8050}$$
$$1.00625^n = 0.3947 + 1$$
$$1.00625^n = 1.3947$$
$$n = \frac{\log 1.3947}{\log 1.00625} \approx 54$$

.....[2]

$\therefore$ The number of years it would take $\dfrac{54}{12} = 4.5$ years.....[0.5]

Question 5

a) Two traders drew a bill amount of Nu 50,000 on 5th September, 2022 due at 6 months. The drawer discounted the bill on 17th December, 2022 at 6% p.a. from a Bill-broker. Find the amount the drawer gets from the Bill-broker. **[3]**

Solution:

Dec:	14 *Days*
Jan:	31 *Days*
Feb:	28 *Days*
Mar:	5 *Days*
Days of grace:	3
Total:	81 *days*.

Given:

Bill Amount $(A) = \text{Nu } 50,000$

Drawn on 5th September, 2022 for 6 months.

Maturity date: 5th March, 2023

Disounted on 17th December, 2022

$n = \dfrac{81}{365},\ i = \dfrac{6}{100} = 0.06$.....[1]

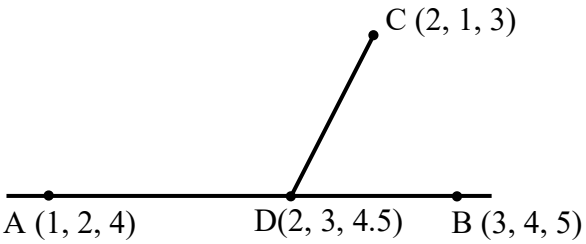
$\therefore$ Banker's discount $= Ani = 50,000 \times \dfrac{81}{365} \times 0.06 = \text{Nu } 665.75$.....[1]

Discounted Value(The amount received by the drawer from the broker)

$= 50,000 - 665.75.$

$= \text{Nu } 49,334.25$.....[1]

b) From the given figure, find the acute angle. **[4]**



Solution:

Direction ratios of BD:

$a_1 = (3-2) = 1,\ b_1 = 4-3 = 1,\ c_1 = 5-4.5 = 0.5$.....[1]

Direction ratios of CD:

$a_2 = 2-2 = 0,\ b_2 = 3-1 = 2,\ c_2 = 4.5-3 = 1.5$.....[1]

$$\left.\begin{aligned} \text{Cos}\,\theta &= \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^{\,2} + b_1^{\,2} + c_1^{\,2}}\sqrt{a_2^{\,2} + b_2^{\,2} + c_2^{\,2}}} \\ \text{Cos}\,\theta &= \frac{(1)(0)+(1)(2)+(0.5)(1.5)}{\sqrt{1^2+1^2+0.5^2}\sqrt{0^2+2^2+1.5^2}} \\ \text{Cos}\,\theta &= \frac{2.75}{1.5\times 2.5} \end{aligned}\right\}$$.....[1]

$$\theta = \text{Cos}^{-1}\left(\frac{2.75}{3.75}\right) = 42.83^o$$.....[1]

Question 6

a) Integrate: $\dfrac{3x+5}{\left(3x^2+10x+2\right)^{2/3}}$ **[3]**

Solution:

$$\left.\begin{aligned} &\text{Let } u = 3x^2+10x+2 \\ &\frac{du}{dx} = (6x+10) \\ &\frac{du}{dx} = 2(3x+5) \\ &\frac{1}{2}du = (3x+5)dx \end{aligned}\right\}$$.....[1]

$$\therefore \int \frac{3x+5}{\left(3x^2+10x+2\right)^{2/3}}dx = \left.\begin{aligned} &\int \frac{1}{2u^{2/3}}du \\ &= \frac{1}{2}\int u^{-2/3}du \\ &= \frac{1}{2}\frac{u^{1/3}}{\frac{1}{3}} + c \end{aligned}\right\}$$.....[1]

$$\left.\begin{aligned} &= \frac{3}{2}\left(3x^2+10x+2\right)^{1/3} + c \quad OR \\ &= \frac{3}{2}\sqrt[3]{3x^2+10x+2} + c \end{aligned}\right\}$$.....[1]

b) The following data shows the sale of two brands of smartphones in percentage, in the United States for two years from the year 2020 to 2022.

Year	**2020**	**2021**				**2022**	
Brands	**4th Quarter**	**1st Quarter**	**2nd Quarter**	**3rd Quarter**	**4th Quarter**	**1st Quarter**	**2nd Quarter**
Apple	65	55	53	47	56	50	?
Samsung	16	27	26	34	22	24	30

Use the regression analysis to predict the number of sales of apple phones in the 2nd quarter of 2022. **[4]**

Solution:

Apple(x)	Samsung(y)	xy	y^2
65	16	1040	256
55	27	1485	729
53	26	1378	676
48	34	1632	1156
56	22	1232	484
53	25	1325	625
330	**150**	**8092**	**3926**

$\sum x = 330,\ \sum y = 150,\ \sum xy = 8092, \sum y^2 = 3926,\ \bar{x} = \frac{330}{6} = 55,\ \bar{y} = \frac{150}{6} = 25$.....[1]

$$b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$
$$b_{xy} = \frac{6(8092) - 330(150)}{6(3926) - 150^2}$$
$$b_{xy} = \frac{-948}{1056}$$
$$b_{xy} = -0.8977$$
.....[1]

$\therefore$ The regression line of x on y is $(x - \bar{x}) = b_{xy}(y - \bar{y})$

$\Rightarrow x - 55 = -0.8977(y - 25)$

$\Rightarrow x = -0.8977y + 22.4425 + 55$

.....[1]

$\Rightarrow x = -0.8977y + 77.4425$.....[0.5]

$\therefore$ When $y = 30,\ x = -0.8977(30) + 77.4425 = 50.5 \approx 50$ *or* 51.....[0.5]

Question 7

a) Find the coordinates of foci, vertices and eccentricity of the curve $9x^2 - 27y^2 = -243$. **[3]**

Solution:

$$9x^2 - 27y^2 = -243$$
$$\frac{9}{-243}x^2 - \frac{27}{-243}y^2 = 1$$
$$\frac{-1}{27}x^2 + \frac{1}{9}y^2 = 1$$
$$\frac{y^2}{9} - \frac{x^2}{27} = 1$$
.....[1]

$a^2 = 9 \Rightarrow a = \pm 3, \quad b^2 = 27 \Rightarrow b = \pm\sqrt{27}$

Eccentricity: $e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{9+27}{9}} = 2$.....[1]

Foci: $(0, \pm ae) = (0, \ \pm 3 \times 2) = (0, \pm 6)$.....[0.5]

Vertex: $(0, \ \pm a) = (0, \pm 3)$.....[0.5]

b) A group of *desuups* have set up a production plant for ground-apple juice with an initial setup cost of Nu 500,000. The additional cost for producing one litre of ground-apple juice is Nu 30, and the demand price of each litre of juice is Nu 80 in the market. During the first month, 5000 litres of juice was produced and sold. **[4]**

(i) Determine the cost function and the revenue function for producing and selling x units of juice.

(ii) Determine the profit function of the company.

(iii) During the first month, what profit or loss has the company incurred?

(iv) If there is a loss in the first month, how many more litres of juice must be sold?

Solution:

Given: $F = 500,000, \ v(x) = 30x$

(i) Cost Function: $C(x) = 500,000 + 30x$.....[0.5]

Revenue Function: $R(x) = 80x$.....[0.5]

(ii) Profit Function: $P(x) = R(x) - C(x)$

$$P(x) = 80x - (500000 + 30x) = 80x - 500,000 - 30x$$
$$P(x) = 50x - 500,000$$
.....[1]

(iii) $P(5000) = 50(5000) - 500,000 = -250,000.$

The company has incurred a loss of Nu 250,000.
.....[1]

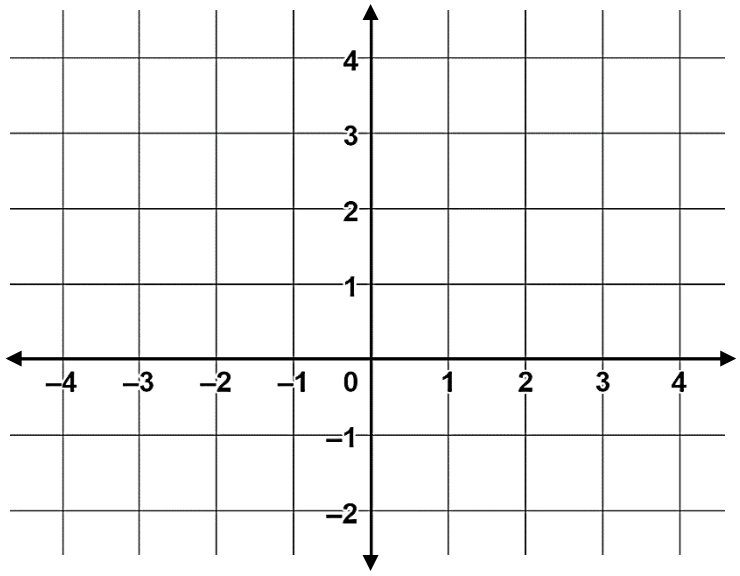
(iv) To avoid the loss, the profit should be 0. i.e. $50x - 500,000 = 0$

$$50x = 500,000$$
$$x = \frac{500000}{50} = 10000.$$

Since 5000 littres are already sold, the company must sell additional 5000 litres
.....[1]

Question 8

a) For the equation $y = 3x^2 + x^3$, find the turning points and sketch the curve on the grid. **[3]**



Solution:

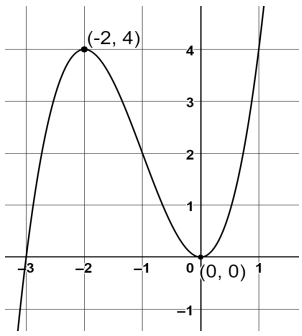
$y = 3x^2 + x^3$

$\frac{dy}{dx} = 6x + 3x^2$

$\frac{dy}{dx} = 0 \Rightarrow 6x + 3x^2 = 0$

$3x(2 + x) = 0$

$x = 0, \quad x = -2.$ .....[1]



….[1] for graph

$\frac{d^2y}{dx^2} = 6 + 6x$

at $x = 0$, $\frac{d^2y}{dx^2} = 6 + 6(0) = 6 > 0, \therefore$ Minimum

Minimum point: (0, 0)

at $x = -2$, $\frac{d^2y}{dx^2} = 6 + 6(-2) = -6 < 0, \therefore$ Maximum

Maximum: (-2, 4)

.....[1]

b) Suppose you are asked to arrange the numbers from 1 to 9, how many numbers can be formed if: **[4]**

(i) all even numbers are always together?

(ii) all even numbers are never together?

(iii) no two even numbers are never together?

(iv) numbers begin with an even and end with an even?

Solution:

(i) There are 4 even numbers and 5 odd numbers.

arrangement of even numbers: 4!

Arrangement of odd numbers: 5!

Total permutations: $4!\times 5!=2880$.....[1]

(ii) Total permutations - number of permutations when all evens are together

$9!-4!\times 5!=362,880-2880=360,000$.....[1]

(iii) $e o e o e o e o e o e$

Since there are 5 odd numbers, they can be arranged in 5! ways.

There are 6 places for the even numbers to be placed so that two even numbers are never together.

Therefore, the require number of permutations: $5!\times {}^{6}P_{4}=43,200$.....[1]

(iv) First place has 4 choices of even numbers to be placed.

Then the last place has 3 choices of even numbers to be placed.

Then the remaining 7 digits can be arranged in 7! ways.

Therefore, the total permutations: $4\times 3\times 7!=60,480$.....[1]

Question 9

a) Given that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}$, find the value of: **[3]**

(i) $P(A/B)$.

(ii) $P(B/A)$.

(iii) $P(A\cup B)$.

Solution:

Given: $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}$

(i) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{1}{4}\times\frac{3}{1}=\frac{3}{4}$.....[1]

(ii) $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{1}{4}\times\frac{2}{1}=\frac{1}{2}$.....[1]

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

$=\frac{7}{12}$ .....[1]

b) Evaluate: $\int \frac{x^2+x+3}{(x-2)(x+1)}dx$ **[4]**

Solution:

$$\int \frac{x^2+x+3}{(x-2)(x+1)}dx = \int \frac{x^2+x+3}{x^2-x-2}dx$$

$$x^2-x-2\overline{)x^2+x+3}\ \ 1$$
$$x^2-x-2$$
$$\underline{(-)(+)(+)}$$
$$2x+5$$

.....[1]

$$\int \frac{x^2+x+3}{(x-2)(x+1)}dx = \int\left(1+\frac{2x+5}{(x-2)(x+1)}\right)$$

Let $\frac{2x+5}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)}$

$2x+5 = A(x+1)+B(x-2)$

.....[1]

Put $x=-1,\ -2+5=-3B \Rightarrow B=-1$

Put $x=2,\ 4+5=3A \Rightarrow A=3$

.....[1]

$$\therefore \int \frac{x^2+x+3}{(x-2)(x+1)}dx = \int\left(1+\frac{3}{(x-2)}-\frac{1}{(x+1)}\right)dx$$
$$= x+3\log|x-2|-\log|x+1|+c$$

.....[1]

Question 10

a) Naku is paying Nu 10,260 at the end of every month for a sum of loan taken from a bank. If the tenure of his loan is 5 years at 9% p.a., calculate the amount of loan he has taken from the bank. **[3]**

Solution:

Given: $a=10,260,\ i=\frac{0.09}{12}=0.0075,\ n=60$.....[0.5]

$$A = \frac{a}{i}\left[1-(1+a)^{-n}\right]$$
$$= \frac{10260}{0.0075}\left[1-1.0075^{-60}\right]$$
$$= \frac{10260}{0.0075}\left[1-0.6387\right]$$
$$= \frac{10260}{0.0075}\times 0.3613$$
$$= 494,258.40 \text{ or } 494,258.81$$

.....[2]

∴ Naku has taken a loan amount of Nu 494,258.40 .....[0.5]

b) Find $\frac{dy}{dx}$ of the following: **[2]**

(i) $\sqrt{\frac{1}{2}x^2 - 3x}$

Solution:

$$\frac{d}{dx}\left(\sqrt{\frac{1}{2}x^2 - 3x}\right) = \frac{1}{2\sqrt{\frac{1}{2}x^2 - 3x}} \frac{d}{dx}\left(\frac{1}{2}x^2 - 3x\right) \text{.....[0.5]}$$

$$\left.\begin{aligned} &= \frac{1}{2\sqrt{\frac{1}{2}x^2 - 3x}}(x-3) \\ &= \frac{(x-3)}{2\sqrt{\frac{1}{2}x^2 - 3x}} \end{aligned}\right\} \text{.....[1]}$$

(ii) $x = t^4 - 2t, \quad y = 4 - t^2.$

Solution:

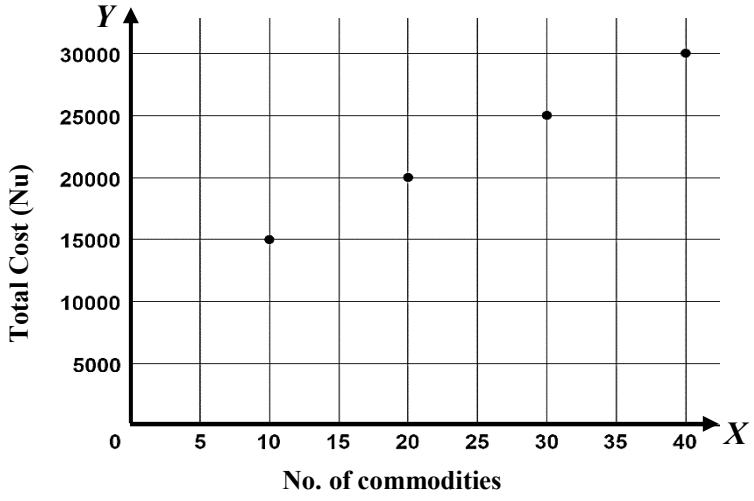
$x = t^4 - 2t, \quad y = 4 - t^2.$

$\frac{dx}{dt} = 4t^3 - 2, \quad \frac{dy}{dt} = -2t$.....[1], ,

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t}{4t^3 - 2} = \frac{-t}{2t^3 - 1}$.....[1]

Question 11

a) The following scatter plot shows the number of commodities produced and their costs incurred: **[3]**



(i) Find the cost function of the commodities.

(ii) Find the marginal cost of the commodities and interpret the result.

Solution:

(i) Given the points: (10, 15000), (20, 20000), (30, 25000), (40, 30000)

Slope of the graph: $\frac{20,000-15,000}{20-10}=\frac{5000}{10}=500$ .....[1]

Since the scatter plots are linear, the equation (Cost function):

$C(x)=500x+b$

Since (30, 25000) lies in the graph,

$C(30)=500(30)+b=25,000$

$b=25000-15,000=10,000$ .....[1.5]

$\therefore$ The cost function $C(x)=500x+10,000$.....[0.5]

(ii) Marginal cost $MC=C'(x)=500$.....[0.5]

The total cost will change by Nu 500 when one more unit is produced.....[0.5]

b) Check the given system for consistency and solve if consistent. **[4]**

$-2x+3y=-7$

$4x-6y=14$

Solution:

$-2x+3y=-7$

$4x-6y=14$

$A=\begin{bmatrix}-2 & 3\\ 4 & -6\end{bmatrix},\ B=\begin{bmatrix}-7\\ 14\end{bmatrix},\ X=\begin{bmatrix}x\\ y\end{bmatrix}$

$|A|=12-12=0,$ therefore, the system is either consisten with many solutions or inconsistent.....[0.5]

$AdjA=\begin{bmatrix}-6 & -3\\ -4 & -2\end{bmatrix}$.....[0.5]

$(AdjA)B=\begin{bmatrix}-6 & -3\\ -4 & -2\end{bmatrix}\begin{bmatrix}-7\\ 14\end{bmatrix}=\begin{bmatrix}42-42\\ 28-28\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix},$

The system is consisten with many solutions. .....[1]

Let $y=k$, where k is any real number

Substitute $y=k$ in the first equation,

$-2x+3k=-7\Rightarrow -2x=-7-3k$

$\Rightarrow x=\frac{7}{2}+\frac{3}{2}k$ or $\frac{7+3k}{2}$ .....[1]

Substituting the value of x and y in the second equation,

$4\left(\frac{7}{2}+\frac{3}{2}k\right)-6k=14$

$LHS: 14+6k-6k$

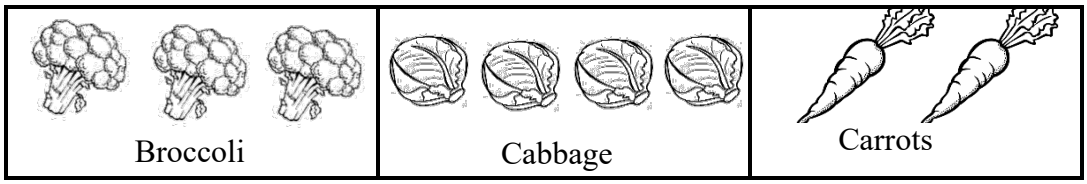
$14=RHS$

Therefore, the system of equations is consistent with many solutions.

$x=\frac{7}{2}+\frac{3}{2}k$ or $\frac{7+3k}{2},\ y=k,$ where, k is any real number. .....[1]

Question 12

a) Following are the types of vegetables available in a vegetable shop, and you want to buy five vegetables in total. **[3]**



(i) Find the number of ways you can buy.
(ii) Find the number of ways you can buy if you want to have minimum of two broccolis.
(iii) Find the number of ways you can buy if you want only two cabbages.

Solution:

(i) The total number of ways to buy 5 vegetables out of 9 is ${}^{9}C_{5} = 126$.....[1]

(ii) The number of ways to buy at least one broccoli:
${}^{3}C_{1} \times {}^{8}C_{4} + {}^{3}C_{2} \times {}^{7}C_{3} + {}^{3}C_{3} \times {}^{6}C_{2} = 330$.....[1]

(iii) The number of ways to buy if no more than two cabbages:
${}^{4}C_{2} \times {}^{5}C_{3} = 60$.....[1]

b) The following data shows the comparison in prices of items in two different hotels. Find the Spearman's rank coefficient of correlation between the prices charged by the hotels. **[4]**

Items	**Pork Curry**	**Ema Datshi**	**Shakam Paa**	**Shakam Datshi**	**Shamu Datshi**	**Sikam Paa**	**Chicken Curry**
Hotel A	150	90	140	120	90	190	90
Hotel B	180	75	130	125	90	180	100

Solution:

Hotel A	**Hotel B**	**R1**	**R2**	**D=R1-R2**	**D^2**
150	180	2	1.5	0.5	0.25
90	75	6	7	-1	1
140	130	3	3	0	0
120	125	4	4re	0	0
90	90	6	6	0	0
190	180	1	1.5	-0.5	0.25
90	100	6	5	1	1
					2.5

……..[1.5] for table

Correction Factors: $\frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \ldots$

$= \frac{1}{12}(3^3 - 3) + \frac{1}{12}(2^3 - 2) = 2 + 0.5 = 2.5$ .....[1]

$r = 1 - \frac{6\left(\sum D^2 + correction\ factor\right)}{n(n^2-1)}$

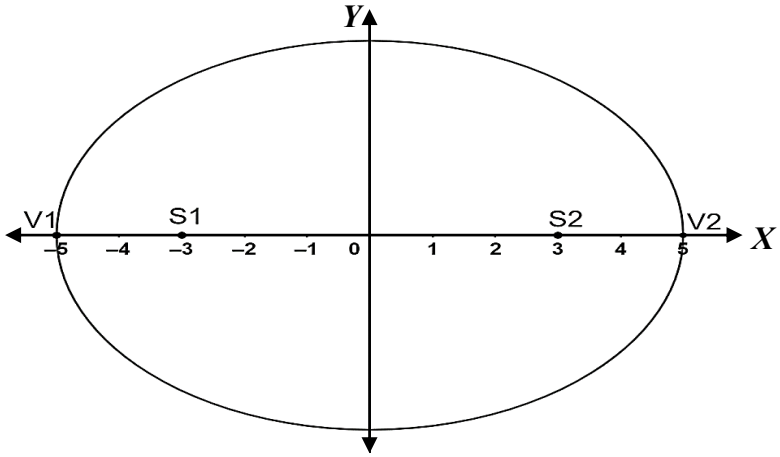
$r = 1 - \frac{6(2.5+2.5)}{7(7^2-1)}$

$r = 1 - \frac{30}{7 \times 48}$ .....[1]

$r = 0.9107$.....[0.5]

Question 13

a) Determine the equation of the conic section shown in the diagram. **[3]**



Solution:

Given: Focus $(\pm ae, 0) = (\pm 3, 0)$, Vertex: $(\pm a, \ 0) = (\pm 5, \ 0)$

$ae = 3 \Rightarrow 5e = 3 \Rightarrow e = \frac{3}{5}$.....[1]

$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \left(\frac{3}{5}\right)^2 = 1 - \frac{b^2}{5^2}$

$\Rightarrow \frac{9}{25} = 1 - \frac{b^2}{25}$

$\Rightarrow \frac{9}{25} = \frac{25 - b^2}{25}$ .....[1]

$\Rightarrow b^2 = 25 - 9$

$\Rightarrow b^2 = 16$

$\therefore$ The equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.....[1]

b) A retailer paid Nu 50,000 on a phone. He wants to give a discount of 20% on the marked price and still wants to make a profit of 30%. **[4]**

(i) What price should be marked on the phone.

(ii) If a customer demands a discount of 30% on the marked price, find the percent profit the retailer will get?

Solution:

(i) Given: Cost Price $= \text{Nu } 50,000.$

Selling Price $= 130\%$ of Cost Price $= 1.3 \times 50,000 = \text{Nu } 65,000.$

Let the marked price be x.

Selling Price $= 80\%$ of the marked price

$65,000 = 0.8x$ .....[1.5]

$$x = \frac{65,000}{0.8} = 81,249.99 \approx \text{Nu } 81,250 \text{.....[0.5]}$$

(ii) Selling Price $= 70\%$ of $81,250 = 0.7 \times 81,250 = \text{Nu } 56,875$

$\therefore$ The retailer's profit $= 56,875 - 50,000 = \text{Nu } 6,875.$ .....[1.5]

$$\% \text{ Profit} = \frac{6875}{50,000} \times 100 = 13.75\% \text{.....[0.5]}$$