

SECTION A [10 MARKS]

Answer **all** questions

Question I

[10]

For each question, there are **FOUR** responses: **A, B, C** and **D**. Choose the corresponding letter of your response and **CIRCLE** it. If you change your answer, cross out the previous response clearly and circle your new answer. **NO** score will be awarded, if you circle more than **ONE** letter.

1. For every matrix **A** there exist a matrix **B** such that $\mathbf{A} + \mathbf{B} = \mathbf{0}$. Which is the matrix **B**?

- A $\mathbf{A}$
- B $\mathbf{A}^T$
- C $-\mathbf{A}$
- D $-\mathbf{A}^T$

2. Solve: $\log_5 x + \log_5 3 = \log_5 30$

- A $x = 27$
- B $x = 10$
- C $x = 5$
- D $x = 3$

3. What is the arithmetic mean of $(x+y)^2$ and $(x-y)^2$?

- A $x^2 - y^2$.
- B $x^2 + y^2$.
- C $x^2 + 2xy + y^2$.
- D $x^2 - 2xy + y^2$.

4. What is the geometric meaning of $|z|$?

- A Real part of z
- B Imaginary part of z
- C Distance from origin in Argand plane
- D Distance of the point representing z from the real axis

5. If $f(x) = (x - 4)^2 - 1$ for $x \in \mathbb{R}$ and $g(x) = \frac{2x+3}{x-2}$ for $x \in \mathbb{R}$. Find $fg(4)$.

- A -5.50
- B -0.33
- C 1.25
- D 2.55

6. What is the adjoint of the matrix $\begin{bmatrix} 2 & -3 \\ 0 & -5 \end{bmatrix}$?

- A $\begin{bmatrix} 2 & 0 \\ -3 & -5 \end{bmatrix}$
- B $\begin{bmatrix} -5 & 0 \\ -3 & 2 \end{bmatrix}$
- C $\begin{bmatrix} -5 & 3 \\ 0 & 2 \end{bmatrix}$
- D $\begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix}$

7. If the discriminant of a quadratic equation is zero, which are the roots?

- A Complex and unequal
- B Complex and equal
- C Real and unequal
- D Real and equal

8. Evaluate: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

- A $2e^{\sqrt{x}} + C$
- B $\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$
- C $\frac{e^{\sqrt{x}}}{2\sqrt{x}} + C$
- D $2\sqrt{x}e^{\sqrt{x}} + C$

9. Find $\frac{dy}{dx}$, if $x = \cos t$ and $y = \sin t$.

- A $\tan t$
- B $\cot t$
- C $-\tan t$
- D $-\cot t$

10. Solve: $e^{2x-3} = 16$

- A 2.773
- B 2.886
- C 2.714
- D 2.102

SECTION B (60 MARKS)

Answer **all** questions

Question II

It is given that the complex number $z_1 = 1 + 2i$.

1. Show that the inequality $|z + z_1 - 2| \leq 3$ can be rewritten in the form $|z - z_0| \leq 3$ and state the coordinates of the complex center z_0 in the form $x + iy$.

[1]

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2. On the complex plane shade the region of points that simultaneously satisfy the following system of inequalities: $|z + z_1 - 2| \leq 3$ and $\operatorname{Re}(z) \geq 2$. [3]

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3. Using your geometric representation from part (2), find the exact maximum possible value of $\arg(z)$ within the shaded regional boundary. [2]

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Question III

The matrix M is defined by $M = \begin{pmatrix} 1 & 1 & a \\ 1 & -3 & 0 \\ 0 & -1 & 1 \end{pmatrix}$, where a is a constant.

1. Given that M is singular, find the value of a .

[2]

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2. a. Given instead that M is non-singular, find M^{-1} .

[3]

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b. Hence solve $\begin{pmatrix} 1 & 1 & a \\ 1 & -3 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, to find x, y and z in terms of a . [2]

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Question IV

A continuous curve is defined by the function:

$$f(x) = 2x^2 - \ln(x^4) \text{ for } x > 0$$

1. Find the derivative $f'(x)$. [2]

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2. Hence determine the stationary points of x where the curve reaches its absolute minimum value. Calculate the exact coordinates of this minima, and confirm its nature by using the second derivative test.

[5]

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Question V

Given that $g(x) = 3x^3 + 2x^2 - 7x + 2$,

1. Show that $(x-1)$ is a factor of $g(x)$.

[2]

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2. Hence solve the inequality $g(x) > 0$.

[5]

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Question VI

It is given that $f(x) = \frac{a^2 - 2ax}{(x-a)(x^2 + 4a^2)}$, where a is a positive constant.

1. Express $f(x)$ in partial fractions.

[3]

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2. Hence find the exact value of $\int_0^a f(x) dx$.

[4]

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Question VII

A straight line described by the linear equation $2x - y + 1 = 0$ intersects a parabolic curve whose equation is given by $y + 2 = x^2$.

1. Determine the exact coordinates of the two distinct points where the line and the parabola intersect by solving these simultaneous equations.

[4]

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Question VIII

1. Consider two independent real-valued functions defined by:

$$f(x) = -x + 3, \quad g(x) = x^2 + 1$$

Construct an explicit expression for the composite function $(g \circ f)(x)$. Expand and simplify your final answer into the standard quadratic form:

$$(g \circ f)(x) = px^2 + qx + r$$

[4]

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Question IX

1. With the help of Argand plane describe the relationship between z and $\bar{z}$. [2]

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2. If $z_1 = 2 + 3i$ and $z_2 = 3 + i$,
a. Represent the complex number $z_1 + z_2$ on the Argand plane. [2]

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b. Show that $|z_1| + |z_2| > |z_1 + z_2|$. **[1]**

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c. Find the polar form of $z_1 + z_2$. **[3]**

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Question X

Consider the function defined by $f(x) = x \ln(x)$ for $x > 0$.

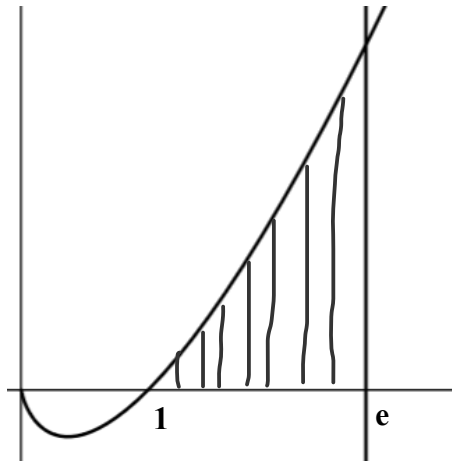
1. Find the general indefinite integral $\int x \ln(x) dx$.

[2]

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2. Using result from part (1), calculate the total area of shaded region shown below.

[2]



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3. This same bounded region is rotated completely through 360 degree (2π radians) about the x –axis. Find the volume of the solid of revolution generated.

[6]

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FORMULAE

Class XI

Strand A: Numbers and Operations

If $y = a^x$, where $a > 0$, $a \neq 1$, and $x > 0$, then $x = \log_a y$

$$\log_a a = 1; \quad \log_a 1 = 0; \quad \log_a a^x = x; \quad a^{\log_a x} = x$$

$$\log_a (xy) = \log_a x + \log_a y; \quad \log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y; \quad \log_a (x)^m = m \log_a x$$

$$\log_a \left(\frac{1}{x} \right) = -\log_a x$$

If $y = e^x$ then $x = \ln y$

Strand B: Patterns and Algebra

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + \binom{n}{n} b^n$$

$$\binom{n}{r} = \frac{n \times (n-1) \times (n-2) \times \dots \times (n-r+1)}{r \times (r-1) \times (r-2) \times \dots \times 3 \times 2 \times 1}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}; \quad \binom{n}{0} = 1; \quad \binom{n}{1} = n; \quad \binom{n}{n} = 1$$

$$nth \text{ term} = a + (n-1)d$$

$$S_n = \frac{n}{2}(a+l) \text{ or } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$nth \text{ term} = S_n - S_{n-1}$$

$$nth \text{ term} = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \text{ or } S_n = \frac{a(r^n-1)}{r-1}$$

$$S_\infty = \frac{a}{1-r}, \text{ where } |r| < 1.$$

$$\frac{d}{dx}(x^n) = nx^{n-1}; \quad \frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$

$$\frac{d}{dx}[kf(x)] = k \frac{d}{dx}[f(x)]; \quad \frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u \frac{du}{dx} + v \frac{dv}{dx}; \quad \frac{d}{dx}\left(\frac{u}{v}\right) = v \frac{du}{dx} - u \frac{dv}{dx}$$

If the value of $\frac{dy}{dx}$ at point (x_1, y_1) is m then

The equation of tangent at that point is $y - y_1 = m(x - x_1)$

The equation of the normal at that point is $y - y_1 = -\frac{1}{m}(x - x_1)$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \text{ where } c \text{ is constant and } n \neq -1.$$

$$\int kf(x)dx = k \int f(x)dx, \text{ where } k \text{ is a constant.}$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c$$

$$x^2 + 2dx = (x+d)^2 - d^2 \quad \text{and} \quad x^2 - 2dx = (x-d)^2 - d^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $f(x) = ax^2 + bx + c$ written in the form $f(x) = a(x-h)^2 + k$,

Line of symmetry: $x = h = \frac{-b}{2a}$

If $a > 0$, there is minimum point at (h, k)

If $a < 0$, there is a maximum point at (h, k)

The discriminant of $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

Class XII

Strand A: Numbers and Operations

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$\text{If } ax^2 + bx + c = 0, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}.A$$

$$X = A^{-1}B$$

For $z_1 = a + bi$ and $z_2 = c + di$:

$$z_1 \pm z_2 = (a \pm c) + (b \pm d)i$$

$$z_1 z_2 = (ac - bd) + (ad + bc)i$$

$$\frac{z_1}{z_2} = \frac{(ac + bd) + (bc - ad)i}{c^2 - d^2}$$

For $z = x + yi$

$$|z| = r = \sqrt{x^2 + y^2}; \quad |z_1 z_2| = r_1 r_2 = |z_1| |z_2|; \quad \left| \frac{z_1}{z_2} \right| = \frac{r_1}{r_2} = \frac{|z_1|}{|z_2|};$$

$$\arg(z) : \tan \theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1} \left| \frac{b}{a} \right|; \quad \arg \left(\frac{z_1}{z_2} \right) = \theta_1 - \theta_2 = \arg z_1 - \arg z_2$$

Polar forms:

Modular-argument form: $(\cos \theta + i \sin \theta)$

Exponential form: $re^{i\theta}$

$$\text{Cube roots of one: } z = 1, z = \frac{-1 + i\sqrt{3}}{2}, z = \frac{-1 - i\sqrt{3}}{2}$$

Strand B: Patterns and Algebra

$$y = x^n, y' = nx^{n-1}$$

$$y = cf(x), y' = cf'(x)$$

$$\text{If } y = uv, \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\text{If } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}e^x = e^x$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sec^2 x dx = \tan x + c$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$$

$$\int uv dx = x \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

$$1 + 2 + 3 + \dots + (n-1) = \frac{1}{2} n(n-1)$$

$$1^2 + 2^2 + \dots + (n-1)^2 = \frac{1}{6} n(n-1)(2n-1)$$

$$1^3 + 2^3 + 3^3 + \dots + (n-1)^3 = \left[\frac{n(n-1)}{2} \right]^2$$

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h \left[\sum_{r=0}^{n-1} f(a+rh) \right]$$

$$A = \int_a^b y dx, V = \pi \int_a^b y^2 dx$$

$$\text{Volume of Cone: } \frac{1}{3} \pi r^2 h$$

$$\text{Volume of Sphere: } \frac{4}{3} \pi r^3$$

$$\text{Volume of Cylinder: } \pi r^2 h$$

$$\text{Volume of Prism: } \text{Base Area} \times \text{height}$$

$$\text{S. Area of Cone: } \pi r l + \pi r^2$$

$$\text{S. Area of Sphere: } 4\pi r^2$$

$$\text{S. Area of closed Cylinder: } 2\pi r h + 2\pi r^2$$

$$\text{Area of sector: } \frac{1}{2} r^2 \theta$$

For a polynomial $P(x)$:

If $P(c) = 0$ then $x - c$ is a factor of $P(x)$; If $P\left(\frac{b}{a}\right) = 0$ then $ax - b$ is a factor of $P(x)$.

If $P(x)$ is divided by $x - c$, the remainder is $P(c)$; If $P(x)$ is divided by $ax - b$, the remainder is $P\left(\frac{b}{a}\right)$.