

SECTION A [10 MARKS]

Answer **all** questions

Question I

[10]

For each question, there are **FOUR** responses: A, B, C and D. Choose the corresponding letter of your response and **CIRCLE** it. If you change your answer, cross out the previous response clearly and circle your new answer. **NO** score will be awarded, if you circle more than **ONE** letter.

1. Which is the equivalent measure in radians of the 120° ?
 - A $\frac{\pi}{3}$.
 - B $\frac{2\pi}{3}$.
 - C $\frac{3\pi}{4}$.
 - D $\frac{5\pi}{6}$.
2. Which of the following is equal to $\cos^4\alpha - \sin^4\alpha$?
 - A $\cos 2\alpha$
 - B $\sin 2\alpha$
 - C $\cos 4\alpha$
 - D $\sin 4\alpha$
3. If $P(A \setminus B) = 1$ and $P(A) \neq 1$, then
 - A Event A and B are independent.
 - B Event A and B are mutually exclusive.
 - C Event B always occurs when A occurs.
 - D Event A always occurs when B occurs.
4. Which of the following is a measure of variation?
 - A Arithmetic Mean
 - B Median
 - C Range
 - D Mode
5. The points P and Q have position vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$. Find the magnitude of the vector $\mathbf{p} + \mathbf{q}$.
 - A $\sqrt{11}$
 - B $\sqrt{43}$
 - C 11
 - D 43

6. The stem and leaf plot below shows the test scores of 15 students.

Stem	Leaf
5	0 2 5
6	1 3 3 8
7	0 4 7 9
8	2 5

What is the median?

- A 74
B 70
C 68
D 63
7. Simplify: $\cos(60^\circ + x) - \cos(60^\circ - x)$
A $-\sin x$
B $-\sqrt{3} \sin x$
C $\cos x$
D $2 \cos x$
8. A data set of 20 values has a mean of 50 and a standard deviation of 8. If every value is increased by 5, what are the new mean and standard deviation?
A Mean = 55, Standard deviation = 8
B Mean = 55, Standard deviation = 13
C Mean = 50, Standard deviation = 13
D Mean = 50, Standard deviation = 8
9. Bhutan Telecom generates 8-digit mobile numbers using the digits 0, 1, 2, 3, ..., 9. If each number starts with 17 and no digit appears more than once, what is the total number of mobile numbers generated?
A 1,000,000
B 151,200
C 40,320
D 20,160
10. The points A (2, 3) and B (8, 7) are the end points of a diameter of a circle. What is the equation of the circle?
A $(x - 5)^2 + (y - 5)^2 = 52$
B $(x - 4)^2 + (y - 5)^2 = 25$
C $(x - 5)^2 + (y - 5)^2 = 13$
D $(x - 4)^2 + (y - 5)^2 = 13$

SECTION B (60 MARKS)

Answer **all** questions

Question II

Given that $\cos \theta = -\frac{12}{13}$ and that θ lies in the third quadrant ($180^\circ < \theta < 270^\circ$).

1.

a. Find the exact value of $\tan \theta$.

[1]

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b. Hence, find the exact value of $\tan 2\theta$.

[2]

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2. a. Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

[3] [2]

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- b. Hence, solve the equation $12\sin x - 5\cos x = 13$ for $0^\circ \leq x < 360^\circ$.

[2]

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Question III

1. The six numbers 1, 1, 2, 3, 5, 5 are to be arranged in a row. Find the number of distinct arrangements possible.

[2]

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The faces of a fair blue die are marked 1,1,2,3,5,5.

The faces of a fair red die are marked 2,3,3,4,4,4.

The two dice are thrown and the scores recorded.

B is the score on the blue die.

R is the score on the red die.

2. The two dice are thrown and the scores recorded. Find the probability that the sum $B + R$ is greater than or equal to 8.

[2]

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3. Define the events:

X : the score on the blue die is odd

Y : the score on the red die is prime

a. Events X and Y are independent. Use suitable calculations to prove this mathematically.

[3]

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b. Determine whether events X and Y are mutually exclusive. Explain your reasoning.

[1]

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Question IV

Relative to the origin, O , the position vectors of points A , and B are

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}.$$

1. Given that $OABC$ is a parallelogram, find the coordinates of C .

[2]

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2. Find the obtuse angle between the diagonals of $OABC$.

[3]

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3. Find the exact perpendicular distance of B from the line through O and A . [4]

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Question V

Given the points $A(2,2,5)$, $B(6,3,-1)$ and $C(x, 5,2)$,

1. Find a vector equation of the line through A and B . [2]

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2. Given that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$, find the value of x . [2]

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3. Hence or otherwise, find the exact area of the triangle ABC . [3]

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Question VI

A teacher wants to investigate the relationship between the number of hours students spend revising (x) and their percentage scores in a final examination (y). A random sample of 6 students gave the following calculation summary.

$$\sum x = 48, \quad \sum y = 390, \quad \sum x^2 = 426, \quad \sum y^2 = 26098, \quad \sum xy = 3296.$$

1. Calculate Karl Pearson's correlation coefficient between x and y . Give your answer correct to 3 significant figures.

[2]

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2.

- a. Find the equation of the regression line of y on x . Give your answer in the form $y = mx + c$.

[2]

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- b. Using the regression line, estimate the examination score for a student who revises for 8 hours. [1]

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3. Comment on the reliability of your answer to (2)(b), based on the value of the correlation coefficient. [1]

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Question VII

1. A box contains 5 red marbles, 3 blue marbles and 2 green marbles. Three marbles are selected at random and not replaced.

- a. Find the probability that exactly two of the selected marbles are red. [2]

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- b. Given that the three selected marbles are not all of the same colour, find the probability that there is at least one blue marble among them.

[5]

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Question VIII

Students applying for colleges under the Royal University of Bhutan (RUB) are required to sit for an entrance examination. Those who are admitted then take a progress test three months after the course has begun. Table 8.1 shows the entrance test scores x and the progress test scores y for a random sample of 8 students.

Table 8.1

x	61	80	74	60	83	92	71	67
y	53	77	61	70	81	54	63	85

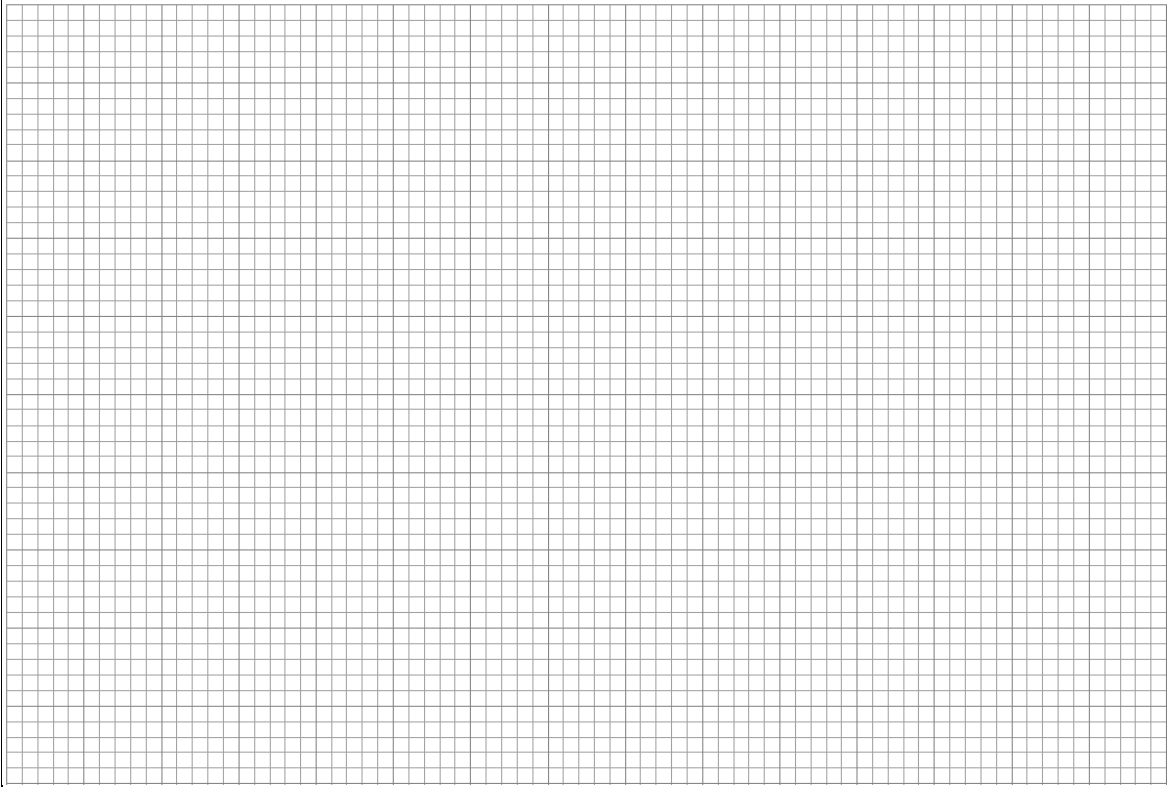
1. Find the correlation coefficient using the rank method.

[3]

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2. Draw a double box-and-whisker plot, using the same scale for both.

[4]

3. State, giving a reason, a conclusion you can make by comparing the box-and-whisker plots.

[1]

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Question IX

1. Find the equation of the perpendicular bisector of the line segment joining A(−5,1) and B (7,2).

[2]

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2. The equations of two regression lines for the same data set are:

$$8x - 10y + 66 = 0$$

$$40x - 18y = 214$$

- a. Identify which line represents the regression of y on x and which represents the regression of x on y . Justify your answer.

[3]

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- b. Find the values of $\bar{x}$ and $\bar{y}$. Show your full working.

[2]

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FORMULAE

Class XI

Strand C: Measurement and Geometry

$$\text{Arc length} = r\theta$$

$$\text{Area of section} = \frac{1}{2}r^2\theta$$

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}; \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta \equiv 1$$

$$\sec^2 \theta - \tan^2 \theta \equiv 1$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$$

$$\text{Midpoint of two points: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Distance between two points: } \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Gradient of a line: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of lines:

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

Equation of circle:

$$(x - a)^2 + (y - b)^2 = r^2, \text{ Centre: } (a, b), \text{ Radius: } r$$

$$x^2 + y^2 + 2gx + 2fy + c = 0, \text{ Centre: } (-g, -f), \text{ Radius: } \sqrt{g^2 + f^2 - c}$$

Strand D: Data and Probability

$$\bar{x} = \frac{\sum x}{n}; \bar{x} = \frac{\sum fx}{\sum f}; \bar{x} = \frac{\sum (x-b)}{n} + b; \bar{x} = \frac{\sum f(x-b)}{\sum f} + b$$

$$\bar{x} = \frac{1}{a} \left[\frac{\sum ax-b}{n} + b \right]; \bar{x} = \frac{1}{a} \left[\frac{\sum f(ax-b)}{\sum f} + b \right]$$

$$\text{IQR} = Q_3 - Q_1;$$

$$Q_1 = \left(\frac{n}{4} \right)^{\text{th}} \text{ or } \left(\frac{\sum f}{4} \right)^{\text{th}} \text{ value}; Q_2 = \left(\frac{n}{2} \right)^{\text{th}} \text{ or } \left(\frac{\sum f}{2} \right)^{\text{th}} \text{ value}; Q_3 = \left(\frac{3n}{4} \right)^{\text{th}} \text{ or } \left(\frac{3\sum f}{4} \right)^{\text{th}} \text{ value}$$

$$\text{Var}(x) = \frac{\sum (x-\bar{x})^2}{n} \text{ or } \frac{\sum x^2}{n} - \bar{x}^2 \text{ or } \frac{\sum (x-b)^2}{n} - \left(\frac{\sum (x-b)}{n} \right)^2$$

$$\text{or } \frac{1}{a^2} \left[\frac{\sum (ax-b)^2}{n} - \left(\frac{\sum (ax-b)}{n} \right)^2 \right]$$

$$\text{Var}(x) = \frac{\sum f(x-\bar{x})^2}{\sum f} \text{ or } \frac{\sum fx^2}{\sum f} - \bar{x}^2 \text{ or } \frac{\sum f(x-b)^2}{\sum f} - \left(\frac{\sum f(x-b)}{\sum f} \right)^2$$

$$\text{or } \frac{1}{a^2} \left[\frac{\sum f(ax-b)^2}{\sum f} - \left(\frac{\sum f(ax-b)}{\sum f} \right)^2 \right]$$

$$\text{SD}(X) = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} \text{ or } \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\text{SD}(X) = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} \text{ or } \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

$$\text{Combined mean of } x \text{ and } y \text{ is } \frac{\sum x + \sum y}{n_x + n_y}$$

$$\text{Combined variance of } x \text{ and } y \text{ is } \frac{\sum x^2 + \sum y^2}{n_x + n_y} = \left(\frac{\sum x + \sum y}{n_x + n_y} \right)^2$$

$$P(A) + P(\text{not } A) = 1 \text{ or } P(A) + P(A') = 1$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B), \text{ when } A \cap B = \phi$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \text{ when } A \cap B \neq \phi$$

$$P(A \text{ or } B \text{ or } C \text{ or } \dots) = P(A) + P(B) + P(C) + \dots \text{ when } A \cap B \cap C \cap \dots = \phi$$

If $A, B, C, \dots$ are independent events, then

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B \cap C \dots) = P(A) \times P(B) \times P(C) \times \dots$$

Class XII

Strand C: Measurement and Geometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

$$\sin 2A = 2 \sin A \cos A; \quad \sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\cos 2A = \cos^2 A - \sin^2 A; \quad \cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$$

$$= 1 - 2 \sin^2 A; \quad = 1 - 2 \sin^2 \frac{A}{2}$$

$$= 2 \cos^2 A - 1; \quad = 2 \cos^2 \frac{A}{2} - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}; \quad \tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha) \text{ and } a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha),$$

$$\text{where } R = \sqrt{a^2 + b^2}, \tan \alpha = \frac{b}{a} \text{ and } 0^\circ < \alpha < 90^\circ.$$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\overrightarrow{CA} = -\overrightarrow{AC}$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

Strand D: Data and Probability

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}} \text{ or } \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$r = 1 - \frac{6\left(\sum D^2 + \text{correction factors}\right)}{n(n^2 - 1)}, \text{ where correction factor: } \frac{1}{12}(m^3 - m) + \dots$$

$$r = \pm \sqrt{b_{yx} b_{xy}}$$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}; \quad b_{xy} = r \frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$y - \bar{y} = b_{yx} (x - \bar{x}); \quad x - \bar{x} = b_{xy} (y - \bar{y})$$

For n distinct objects, there are:

$${}^n P_n = n! = n(n-1)(n-2)\dots(3)(2)(1), \text{ for all } n \text{ objects.}$$

$${}^n P_r = \frac{n!}{(n-r)!} \text{ permutations of } r \text{ objects.}$$

$$\frac{n!}{p! \times q! \times r! \times \dots} \text{ permutations in which there are } p, q, r, \dots \text{ of each type.}$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \text{ combinations of } r \text{ objects.}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

$$P(A) \times P(B | A) = P(B) \times P(A | B)$$

$$P(A) + P(\bar{A}) = 1$$