

SECTION A [10 MARKS]

Answer **all** questions

Question I

For each question, there are **FOUR** responses: **A**, **B**, **C** and **D**. Choose the corresponding letter of your response and **CIRCLE** it. If you change your answer, cross out the previous response clearly and circle your new answer. **NO** score will be awarded, if you circle more than **ONE** letter.

[10]

1. Which statement is true about the equation $x - 7 = 0$?
 - A It has no solution.
 - B It has two solutions.
 - C It has one solution.
 - D It has infinitely many solutions.

2. Which one of the following sequences is a geometric progression?
 - A 3, 6, 12, 24
 - B 2, 4, 8, 12
 - C 1, 2, 3, 5
 - D 2, 0, 2, 0

3. Find $\int x(x + 1) \, dx$.
 - A $x^2 + x + c$
 - B $2x + x + c$
 - C $x^3 + x^2 + c$
 - D $\frac{x^3}{3} + \frac{x^2}{2} + c$

4. How many terms are there in the expansion of $(x + 2y^2)^7$?
 - A 8
 - B 9
 - C 10
 - D 14

5. Which of the following is a correct expression for $\frac{d}{dx} \left(\frac{2x^{\frac{9}{2}} + 10}{x^3} \right)$?

A $3x^{\frac{3}{2}}$

B $\frac{6}{5}x^{\frac{5}{2}} - \frac{5}{x^2}$

C $3x^{\frac{3}{2}} + \frac{10}{x^3}$

D $\frac{9}{2}x^{\frac{1}{2}} - \frac{30}{x^4}$

6. What is the factored form of $2x^2 - 8x + 6$?

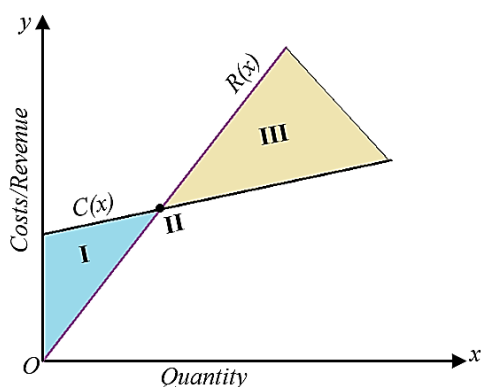
A $2(x + 3)(x - 1)$

B $(x - 4)(x + 3)$

C $2(x - 3)(x - 1)$

D $(x - 3)(x - 1)$

7. The graphs of total cost function $C(x)$ and total revenue function $R(x)$ are as shown in the diagram. Identify the parts labelled.



A I-total loss, II-break-even point, III-total profit

B I-total profit, II-break-even point, III-total loss

C I-total revenue, II-break-even point, III-total cost

D I-total revenue, II-break-even point, III-total revenue

8. Which is the domain of $f(x) = \frac{1}{x^2 - 9}$?

A $x > 3$

B $x \neq \pm 3$

C $x < -3$

D $x \geq 3$

9. If the total cost function is $C(x) = 500 + 20x + x^2$, what is the marginal cost when $x = 5$?
- A 625
B 600
C 40
D 30
10. The polynomial $f(x)$ has $(x - 3)$ as a factor. Which one of the following statements about $f(x)$ must be true?
- A $f(0) = 3$
B $f(3) = 1$
C $f(-3) = 0$
D $f(3) = 0$

SECTION B (60 MARKS)

Answer **all** questions

Question II

An absolute value profile satisfies the mathematical statement $|2x - 3| = x + 4$.

1. Separate this modulus expression into its two constituent linear equations representing its positive and negative arguments.

[2]

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2. Solve both linear equations from part (1) and find two possible solutions for x . [2]

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3. Hence find the complete set of solutions of the inequality $|2x - 3| \leq x + 4$. [2]

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Question III

A curve has the equation $y = 2x^3 - 9x^2 + 12x + 5$.

1. Find an expression for $\frac{dx}{dy}$.

[2]

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2. Using your answer from part (1), find the x –coordinates of the two stationary points on the curve.

[2]

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3. Find the expression for $\frac{d^2y}{dx^2}$, and hence determine the nature of each stationary point found in part (2). [2]

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4. Find the equation of the tangent to the curve at the point where the curve crosses the y -axis. Give your answer in the form $y = mx + c$. [3]

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Question IV

A corporate enterprise models its Total Cost (C) and Total Revenue (R) for producing and selling x units of a commercial commodity (in thousands of Ngultrum) using the following functions:

$$C(x) = x^3 - 6x^2 + 15x + 20$$

$$R(x) = 27x$$

1. Find an expression for the Average Cost function, $AC(x)$, and determine the production level x where the Average Cost is minimized.

[3]

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2. Construct the Total Profit function, $P(x) = TR(x) - TC(x)$, expressing it in simplified polynomial form.

[1]

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3. Find the Marginal Profit function, $MP(x) = \frac{dP}{dx}$ and calculate the positive level of output x that yields maximum total profit. [3]

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4. Economists state that at the point of maximum profit, Marginal Revenue (MR) must equal Marginal Cost (MC). Verify this commercial identity by evaluating both MR and MC at the profit-maximizing output level found in part (3). [2]

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Question V

Consider the rational function defined by

$$f(x) = \frac{5x-3}{x^2-x-2}$$

1. Express $f(x)$ in partial fractions.

[3]

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2. Using your result from part (1), find the exact expression for the indefinite integral $\int f(x) \, dx$.

[2]

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3. Find the exact area of the region completely bounded by the curve $y = f(x)$, the x -axis, and the vertical lines $x = 3$ and $x = 4$. Express your final answer as a single logarithm of the form $\ln(k)$, where k is a rational number.

[4]

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Question VI

Consider the function $f(x) = |2x - 4|$ defined for all real values of x .

1. Sketch the graph of $y = f(x)$. Clearly label the coordinates of the vertex and the y -intercept. [2]

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2. A straight line has the equation $y = x + 2$. On your sketch from part (1), add the graph of this line and find algebraically the x -coordinates of the points where the line intersects the modulus graph [2]

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3. Hence, deduce the exact solution set for the modulus inequality and shade the solution on the graph:

[2]

$$|2x - 4| \leq x + 2$$

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Question VII

The polynomial function $P(x)$ is given by $P(x) = x^3 - x^2 + kx + 6$, where k is a constant.

1. Given that when $P(x)$ is divided by $(x + 1)$ the remainder is 8, use the remainder theorem to show that $k = -5$.

[2]

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2. Using the value of k from part (1), use the factor theorem to show that $(x + 2)$ is a linear factor of $P(x)$.

[1]

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3. By polynomial division, express $P(x)$ completely as a product of three linear factors.

[3]

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Question VIII

A curve has the equation $y = f(x)$, defined for $x > 0$.

It is given that the second derivative of the curve is:

$$\frac{d^2y}{dx^2} = 6x - 8$$

1. Find the general expression for the gradient function, $\frac{dy}{dx}$, in terms of x and an arbitrary constant C_1 . [2]

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2. Given that the curve has a stationary point at $(2, 1)$, find the value of C_1 and find the specific expression for the gradient function $\frac{dy}{dx}$. [2]

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3. Find the explicit equation of the curve, $y = f(x)$, using the fact that the curve passes through the stationary point (2, 1).

[2]

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Question IX

A curve has the equation

$$x^2 - 4xy + 2y^2 + 18 = 0$$

1. Use implicit differentiation to show that the gradient of the curve is given by:

[3]

$$\frac{dy}{dx} = \frac{2y - x}{2y - 2x}$$

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2. Find the coordinates of the two stationary points on the curve where $\frac{dy}{dx} = 0$. [3]

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3. By differentiating your expression from part (1) implicitly a second time, show that the higher -order derivatives satisfy the relationship: [3]

$$\frac{d^2 y}{dx^2} = \frac{y}{(y-x)^3}$$

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FORMULAE

Class XI

Strand B: Patterns and Algebra

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

$$|a| = |b| \Leftrightarrow a^2 = b^2$$

$$|a| > |b| \Leftrightarrow a^2 > b^2$$

$$|a| < |b| \Leftrightarrow a^2 < b^2, \text{ if } b \neq 0.$$

$$ax^2 + bx + c = 0, (a \neq 0)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d$$

$$S = \frac{n}{2}(a+l)$$

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$n^{\text{th}} \text{ term} = S_n - S_{n-1}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} (r \neq 1)$$

$$S_n = \frac{a-lr}{1-r} = \frac{lr-a}{r-1}$$

$$A.M = \frac{a+b}{2}, \text{ if } a \text{ and } b \text{ are the given numbers}$$

$$G.M = \sqrt{ab}$$

$$\frac{dy}{dx} \text{ or } f'(x) = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} (ax+b)^n = n(ax+b)^{n-1} \times a$$

$$\frac{d}{dx} (uv) = u \frac{d}{dx} v + v \frac{d}{dx} u$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a} + c, n \neq -1$$

$$\int_a^b f(x) dx = [F(x)]_a^b = [F(b) - F(a)]$$

Class XII

Strand B: Patterns and Algebra

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

$$|a| = |b| \Leftrightarrow a^2 = b^2$$

$$|a| > |b| \Leftrightarrow a^2 > b^2$$

$$|a| < |b| \Leftrightarrow a^2 < b^2, \text{ if } b \neq 0.$$

$$|x - a| < b \Leftrightarrow a - b < x < a + b$$

If $P(x)$ is divided by $D(x)$, then

$$P(x) = D(x).Q(x) + R(x)$$

Where: $Q(x)$ is quotient

$R(x)$ is Remainder

$(x - a)$ is factor of $f(x)$ if and only if $f(a) = 0$.

Conversely: if $f(a) = 0$, then $(x - a)$ is a factor.

If a polynomial $P(x)$ is divided by $x - c$,
the remainder is $P(c)$.

If a polynomial $P(x)$ is divided by $ax - b$,
the remainder is $P\left(\frac{b}{a}\right)$.

$$a^2 - b^2 = (a + b)(a - b)$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

In the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = x^n, y' = nx^{n-1}$$

$$y = (ax + b)^n, y' = n(ax + b)^{n-1} \times a$$

$$\text{If } y = u \pm v, \text{ then } \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\text{If } y = uv, \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\text{If } y = \frac{u}{v}, \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$C(x) = F + V(x), A(x) = \frac{C(x)}{x}$$

$$R(x) = p.x, P(x) = R(x) - C(x)$$

$$MC = \frac{d}{dx}(C(x)), MR = \frac{d}{dx}(R(x))$$

$$C = C(x) = \int MC \, dx + k$$

$$R = R(x) = \int MR \, dx + k$$